Kahan discretisation of a cubic Hamiltonian system

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Abstract: An important criterion for an equation or system of equations in the complex plane to be integrable is the Painlevé property, stating that all movable singularities of all solutions of the equation are poles. The six Painlevé equations are essentially the only non-linear equations in the class of second-order rational ODEs with this property, the solutions of which define new transcendental functions. They are equivalent to certain Hamiltonian systems with polynomial Hamiltonians. An analogue of the Painlevé property for discrete equations is known as singularity confinement. When passing from a discrete Painlevé equation to a differential equation, which can be achieved by taking an appropriate continuous limit, integrability is preserved. Going the other direction, known as discretisation, is not as straightforward. We discuss a discretisation method due to Kahan applied to a certain cubic Hamiltonian system, leading to a discrete system which possesses a sort of partial singularity confinement. This also manifests itself in the value for the algebraic entropy of this system, a measure of complexity for the iterates under the discrete map, which, although non-zero, is smaller than expected for a generic, non-integrable discrete mapping. Joint work with Galina Filipuk.

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