

Rank distributions, nonlinear dynamics, and number theory

Alberto Robledo, Carlos Velarde

Abstract: We address the wide spectrum physics of ranked data originating from many different sources that include those obeying the laws of Zipf and Benford but incorporate other types of behaviors. The formalism distinguishes between data sorted out by magnitude or by frequency and can be expressed in two equivalent ways, one stochastic and the other deterministic. We show that rank distributions fall into distinct universality classes each with links to number theory sets such as factorial, natural, prime and Fibonacci numbers. We reproduce these sets of numbers, exactly or increasingly accurately, as specific trajectories of iterated maps. Subsequently we obtain their reciprocals, or algebraic inverses, also as trajectories of associated nonlinear maps and calculate their sums or series. All cases are contained in the renormalization group fixed-point map for the tangent bifurcation each for a specific value of the nonlinearity z . The value $z=2$ is crucial in different ways as it is a borderline power. For $z>2$ the series of reciprocals converges, while for $z\leq 2$ diverges. At $z=2$ logarithmic corrections arise in correspondence with prime number known bounds. Also, $z=2$ signals the vanishing of the contraction dimension $Z=2-z$, the minimal value that an attractor can exert on ensembles of trajectories. Furthermore, $z=2$ corresponds to the classical case of Zipf law. Rescaled reciprocals of number sets link to rank distributions, while rescaled number sets provide probability densities that have extensive generalized entropies as measured by maximum rank values.

-
- ¹⁾ Alberto Robledo, Ph.D.: Instituto de Fisica, Universidad Nacional Autonoma de Mexico, Apartado Postal 20-364, 01000, Ciudad de Mexico, Mexico (MX), robledo@fisica.unam.mx, the author presented this contribution at the conference in the special session: "A special session dedicated to Prof. Miguel A.F. Sanjuán on the occasion of the celebration of his 60th anniversary" organized by J. Awrejcewicz.
- ²⁾ Carlos Velarde, Ph.D.: Instituto de Investigaciones en Matematicas Aplicadas y en Sistemas, Universidad Nacional Autonoma de Mexico, Apartado Postal 20-364, 01000, Mexico City, Mexico (MX), velarde@unam.mx.