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Algorithm for suboptimal feedback construction based on Pade approximation for nonlinear control problems

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Abstract: On several classes of nonlinear dynamic problems with a parameter the possibility for constructing parametric families of solutions that arise in control theory on the basis of the Padé approximation (PA) is shown using the asymptotic expansions for small and large values of the parameter and finite-dimensional optimization algorithms and also by taking into account the characteristics of the asymptotics on which the PA is based. The possibility for increasing the accuracy of approximations and the enhancement of the interpolation and extrapolation properties of one-point and two-point PAs in comparison with asymptotic approximations is demonstrated.

Keywords: SDRE, finite time interval, parameter, small time step, Pade approximation

1. Asymptotic expansion of the Riccati equation solution on a finite time interval

In the SDRE approach, a large role is played by the differential matrix Riccati equations [1-3] for a finite interval or matrix algebraic Riccati equations with state-dependent coefficients for infinite interval. If the control system contains small or large parameters the synthesis can be constructed using asymptotic expansions by these parameters and the operators based on them (for example, Pade approximations) [4]. For example, consider the following class of nonlinear controlled systems without control constraints

$$\begin{gathered}\dot{x} = A(x)x + \varepsilon B(x)u,\ x(0) = x^0, \\ x(t) \in X \subset R^n,\ u \in R^r,\ t \in [0,T],\ \varepsilon \in (0,\infty),\end{gathered} \tag{1}$$

$$J(u) = \frac{1}{2}x^T(T)Fx(T) + \frac{1}{2}\int_0^T (x^T(t)Q(x,\varepsilon)x(t) + u^T(t)Ru(t))dt, \tag{2}$$

where $X \subset R^n$ is a bounded set. In this case the control is chosen in the formally linear feedback form but with state dependent coefficients $u(x) = -\varepsilon R^{-1}B^T(x)P(x,t,\varepsilon)x(t)$, where $P(x,t,\varepsilon)$ is a solution of the modified matrix differential Riccati equation for all $x \in X$ and $\varepsilon \in (0,\infty)$.

$$\begin{gathered}dP/dt = -P(x,t,\varepsilon)A(x) - A(x)^T P(x,t,\varepsilon) + \varepsilon^2 P(x,t,\varepsilon)B(x)R^{-1}B^T(x)P(x,t,\varepsilon) - \\ -Q(x,\varepsilon),\ P(x,t,\varepsilon)\big|_{t=T} = F,\ F > 0.\end{gathered} \tag{3}$$

The asymptotic of $P(x,t,\varepsilon)$ in (3) for small ε is constructed as a regular power series $\tilde{P}_2(x,t,\varepsilon)=\tilde{P}_0(x,t)+\varepsilon\tilde{P}_1(x,t)+\varepsilon^2\tilde{P}_2(x,t)$ [5], and for $\varepsilon\to\infty$ after the replacement $\varepsilon=\frac{1}{\mu},\mu\to 0$ the asymptotic approximation is constructed as a partial sum of the regular and boundary series by μ [6] $P(x,t,\mu)=\hat{P}_2(x,t,\mu)+\Pi_2\,P(x,\tau,\mu),\ \tau=\frac{t-T}{\mu^2}\le 0$. Then the two-point PA [2/2] is constructed in the form $PA_{[1/2]}(x,t,\tau,\varepsilon)=(\mathrm{M}_0(x)+\varepsilon\,\mathrm{M}_1(x)+\varepsilon^2\,\mathrm{M}_2(x)+\Pi M_0(x,\tau)+\varepsilon\,\Pi M_1(x,\tau)+\varepsilon^2\,\Pi M_2(x,\tau))\times$ $\times\left(E+\varepsilon\,\mathrm{N}_1(x)+\varepsilon^2\mathrm{N}_2(x)\right)^{-1}$. By simultaneously equating the two constructed asymptotic expansions with this representation and then comparing the coefficients with the same powers of the parameter we obtain a system of equations for determining the unknown coefficients of the PA [4].

2. Algorithm and numerical experiments

Using the Pade approximation of the feedback gain matrix as a basic framework all the unknown feedback control elements are approximated by segments of expansions in orthogonal systems, which are selected from the minimum of the functional (2). Experiments show that for the same calculation accuracy the algorithm is more efficient in terms of the calculation time for the construction of parametric families of regulators.

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