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The asymptotic solutions of the boundary value problem of convective diffusion around drops with volumetric nonlinear chemical reaction.

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Abstract:: We consider a stationary problem of convective diffusion around a droplet, which is streamlined by a liquid flow at low Reynolds numbers, taking into account a nonlinear volumetric chemical reaction. The characteristic feature of the problem is the presence of two dimensionless parameters: a constant of rate of the volumetric chemical reaction k_v, and Peclet number Pe which determine the concentration distribution in the flow. The quantity constant of rate of the volumetric chemical reaction k_v and Peclet number Pe assumed to have a constant value. It is a boundary value problem for a quasilinear partial elliptical equation with a small parameter multiplying in higher derivatives. Small parameter corresponds to large Peclet numbers. The limiting equation, when the small parameter is equal to zero, has singular points of the saddle type. Several boundary layers appear outside the drop. The matching conditions for solutions are formulated at the boundaries between neighboring areas. The principal terms of the asymptotics of the solution are constructed around the drop.

Keywords: asymptotic expansions, convective diffusion, matching method, Peclet number.

1. Introduction

Let us consider stationary diffusion near a spherical droplet of radius a, in a translational flow of a viscous incompressible fluid with a velocity U at infinity at low Reynolds numbers Re, taking into account the bulk chemical reaction. The concentration distribution in dimensionless variables satisfies the boundary value problem (see, for example, [1], Ch. 5, (6.1) - (6.3))

$$\Delta U = Pe(\overline{V},\nabla)\cdot U + k_v F(U), \tag{1}$$

$$U = 1 \text{ at } r = 1\text{; } U \to 0 \text{ when } r \to \infty, \tag{2}$$

where

$$\overline{V} = (V_r, V_\theta, 0), V_r = \frac{1}{r^2 sin\theta}\frac{\partial\psi}{\partial\theta}, V_\theta = -\frac{1}{r sin\theta}\frac{\partial\psi}{\partial r}, \tag{3}$$

$$\psi(r,\theta) = (r-1)\left(2r - \frac{\lambda}{\lambda+1}\left(1+\frac{1}{r}\right)\right)sin^2\theta/4, \tag{4}$$

where the Peclet number Pe = aU / D, where D is the diffusion coefficient of the substance in the external phase, kv is the rate constant of the bulk chemical reaction, Δ is the Laplace operator, ∇ is the Hamilton operator. In a spherical coordinate system r, θ with the origin at the center of the drop (the angle θ is measured from the direction of flow at infinity), the velocity field of the liquid outside the spherical drop is determined from the expressions (3) (see [1], Ch. 3), where ψ (r, θ) is the stream function outside the spherical drop and has the form (4) λ is the ratio of the viscosity of the drop to the viscosity of the environment.

2. Results and Discussion

It is assummed that F(U) is continuous and

$$F: R^1 \rightarrow R^1,\ F(0) = 0,\ 0 < F'(U), \tag{5}$$

$$F(u) = u + F_2u^2 + F_3u^3 + \cdots + F_ku^k + O(u^{k+1}) \tag{6}$$

Problems analogous to (1), (2) and a broader class of problems, were considered in [1]. In the absence of chemical reaction (i.e., with $k_v = 0$), problem (1), (2) was analyzed in [1-3] by the method of matched asymptotic expansions [4].

3. Concluding Remarks

It is shown in the work that in the vicinity of the rear critical point the solution of the problem is essentially weakly nonlinear. This affects the nature of mass transfer in the wake of the drop (in the inner boundary).

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