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Identification of the model parameters based on the ambiguous branches of resonance response curves

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Abstract: A concept of the method of determining the parameters describing the damping and the nonlinearity, which can be of physical or geometrical nature, is presented in the paper. The main idea is explained regarding mechanical systems with one degree of freedom, however, the method can be also employed to identification for systems with two DoF provided that the couplings are weak and the resonances do not occur simultaneously. The analysis of stationary resonance states can be reduced only then to the third-degree equation, which is a necessary condition for the applicability of this method. Numerical simulations, which are carried out, confirm the usefulness and accuracy of the method.

Keywords: Duffing's equation, multiple scales method, main resonance, resonance response curves

1. Introduction

Many mechanical systems of one degree of freedom with the nonlinearity of the cubic type and the viscous damping are governed by the Duffing equation of the form [1]

$$\ddot{x}(\tau) + \alpha\dot{x}(\tau) + x(\tau) + \beta x(\tau)^3 = f_0 \cos(p\,\tau), \tag{1}$$

supplemented with the following initial conditions

$$x(0) = x_0, \;\; \dot{x}(0) = v_0, \tag{2}$$

where all quantities are dimensionless, and τ is the time, α, β – parameters to be identified, f_0, p – the amplitude and frequency of the harmonic forcing.

2. Idea of Identicication

Employing the method of multiple scales in the time domain [2-3], one can obtain the approximate solution to the problem given by Eqs (1) – (2). An important part of the solving procedure is the problem of determining the amplitudes and phases. The introduction of several variables $\tau_0, \tau_1, \tau_2, \ldots$ describing the evolution of the system over time allows one to isolate the problem as an independent one. The changeability of the generalised coordinates with the frequencies of order the mechanical system eigenfrequencies is described using the fast scale τ_0. The other variables are destined to describe the slow change of the amplitude and phases. In turn, the modulation equations give the possibility to study the periodic stationary vibration at the main resonance. The amplitude of the periodic stationary vibration satisfies the following equation

$$16\alpha a_1^2 + (3\alpha a_1^2 - 8s)^2 a_1^2 - 16 f_0^2 = 0, \tag{3}$$

where s is the detuning parameter, i.e. it is assumed that $p = 1 + s$. Regarding Eq. (3) as the equation of the third order with respect to the square of the amplitude a_1, and analysing the sign of the numerator of its discriminant given by

$$N_\Delta = (64s^3 + 144\alpha^2 s - 81\beta f_0^2)^2 + 64(3\alpha^2 - 4s^2)^3, \tag{4}$$

one can determine the area Ω on the plane $s - f_0$ the points of which satisfy the inequality $N_\Delta < 0$. Each point of the area Ω depicts the parameters of the external force that ensure the existence of three distinct real roots of Eq. (3). The boundaries f_l and f_u of the region Ω, shown in Fig. 1, are defined analytically formulae as follows

$$f_u^2 = \frac{64s^3}{81\beta} + \frac{16s\,\alpha^2}{9\,\beta} + \frac{8\sqrt{64s^6 - 144s^4\alpha^2 + 108s^2\alpha^4 - 27\alpha^6}}{81\beta}, \tag{5}$$

$$f_l^2 = \frac{64s^3}{81\beta} + \frac{16s\,\alpha^2}{9\,\beta} - \frac{8\sqrt{64s^6 - 144s^4\alpha^2 + 108s^2\alpha^4 - 27\alpha^6}}{81\beta}. \tag{6}$$

By experimentally determining the resonance response curves, one can find the values s_0 and s_e of the frequency of the harmonic force at which the jumps occur, corresponding to the entry or exit from the zone of ambiguous responses. Then, solving the equations (5) – (6) gives the approximate values of the parameters α and β.

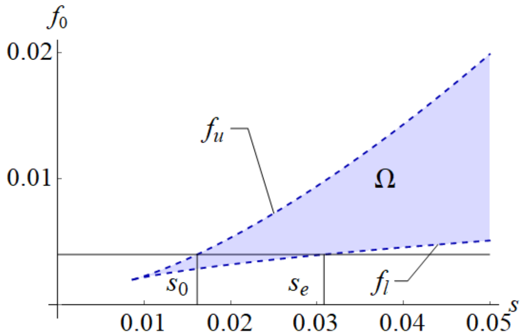

Fig. 1. The region of the ambiguous resonance responses.

The method was initially tested using simulations in which the ambiguous zones of the resonance response curves were determined based on solutions obtained numerically, after confirming that the solutions satisfy the assumptions of the stationary periodic vibration. The results of the tests have been satisfactory, however, it seems advisable to develop methods for estimating the identified parameters based on statistical inference.

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References

[1] NAYFEH A.H, MOOK D.T: *Nonlinear Oscillations.* Wiley&Sons: New York, 1995.

[2] AWREJCEWICZ J, KRYSKO VA, *Introduction to asymptotic methods*. Chapman and Hall: Boca Raton, 2006.

[3] WEINAN E, *Principles of Multiscale Modeling.* Cambridge University Press: Cambridge, 2011.