DSTA 2021 dys-ta.com
16th INTERNATIONAL CONFERENCE
Dynamical Systems – Theory and Applications
December 6-9, 2021, On-line

Multistability in Remote Synchronization Detected via Symbolic Dynamics

JULIANA C. LACERDA[1], CELSO FREITAS[1], ELBERT MACAU[2*]

1. Associated Laboratory for Computing and Applied Mathematics, National Institute for Space Research - INPE, São José dos Campos , Brazil
2. Institute of Science and Technology, Federal University of Sao Paulo, UNIFESP, Brazil
* Presenting Author

Abstract: In this work, we introduce a new approach, based on symbolic dynamics and complex network formalism, to characterize multistability in the remote synchronization phenomena, where the dynamical system is governed by the Stuart-Landau equation and the topology is star-like. This methodology has already been used to detect periodic windows and chaos in nonlinear systems and we show that it is able to detect the regions where multistability takes place and we compare the results obtained by traditional methods.

Keywords: synchronization, multistability, symbolic dynamics

1. Introduction

Two of the most important phenomena in nonlinear dynamical systems are multistability and remote synchronization. In the first, the system may enter in different states after a transient, due to the coexistence of multiple attractors, which implies that the final state has a strong dependence to the initial conditions, in a way that the synchronous state is reached only by a set of these values [1]. In the latter, dynamical units that are not directly connected enter in a synchronous state [2]. In order to quantify the synchronization of dynamical systems, metrics like the order parameter and the partial synchronization index are used [3].

An alternative approach to study the dynamics of nonlinear systems was proposed [4,5]. With this method, instead of using usual metrics to characterize the system, like Lyapunov exponents and bifurcation diagram, metrics from complex networks like mean degree and betweenness centrality is used. This approach makes use of the time series of the system to generate undirected graphs by utilizing symbolic dynamics and then uses the formalism of complex networks to extract information of these graphs and then characterize the system. It was shown that this method is able to detect periodic windows and chaos in the Logistic and Hénon maps [4].

In this work, we introduce a new way to characterize multistability in the phenomena of remote synchronization making use of symbolic dynamics and complex network formalism. Our approach is not able to differ from a synchronous to a non-synchronous state but it is capable of detecting the region where multistability takes place.

2. Results and Discussion

The system is given by a star topology composed of eleven nodes whose dynamics is modelled by the Stuart-Landau equation [6]. This system was studied by [7] and presented multistability in the synchronization of the peripheral nodes in a certain region of coupling. In this work we make use of symbolic dynamics (model DCSD in [4]) to turn the time series of the oscillators into a binary series and, after that, into a decimal series. This decimal series is then converted in networks, where each number corresponds to a node and there exists a connection between two nodes in the network only if they are neighbours in the decimal series. Metrics like betweenness centrality, density and mean degree are then calculated. For a fixed value of coupling, twenty distinct time series, and consequently, twenty distinct graphs are generated for each oscillator. Also, this procedure is done for several values of coupling. The mean and standard deviation of these complex network metrics are then calculated and compared to the usual metric used, called partial synchronization index [3], which is frequently used to quantify remote synchronization. By comparing both methods it is able to verify the existence of a multistability region at the same coupling interval.

3. Concluding Remarks

The results show that the methodology applied in this work is capable of detecting the multistablity region, although outside this region it can not distinguish from the synchronous to the non-synchronous state.

References

[1] U. Feudel. Complex dynamics in multistable systems. International Journal of Bifurcation and Chaos, 18(06):1607-1626, 2008.

[2] A. Bergner, M. Frasca, G. Sciuto, A. Buscarino, E. Ngamga, L. Fortuna, and J. Kurths. Remote synchronization in star networks. Physical Review E, 85(2):026208, 2012.

[3] J. Gomez-Gardenes, Y. Moreno, and A. Arenas. Paths to synchronization on complex networks. Physical review letters, 98(3):034101, 2007.

[4] V. L. Freitas, J. C. Lacerda, and E. E. Macau. Complex networks approach for dynamical characterization of nonlinear systems. International Journal of Bifurcation and Chaos, 29(13):1950188, 2019.

[5] X. Yu, Z. Jia, and X. Jian. Logistic mapping-based complex network modeling. Applied Mathematics, 4(11):1558, 2013.

[6] E. Panteley, A. Loria, and A. El Ati. On the stability and robustness of stuart-landau oscillators. IFAC-PapersOnLine, 48(11):645-650, 2015.