

# Bifurcation analysis of nonlinear piezoelectric vibration energy harvester

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**Abstract:** This paper deals with an analysis of nonlinear piezoelectric vibration energy harvesting system, which is capable to generate electrical power from mechanical vibrations. A kinetic energy of vibration is transferred into useful electricity by a strain of piezoelectric layers on an oscillated cantilever. A nonlinear stiffness is created by additional magnets with separation distance. The presented work is concerned with an analysis of this nonlinear magneto-elastic system of piezoelectric vibration energy harvester by use of bifurcation diagrams, where magnet separation distance acts as a control parameter. A response of this energy harvester and harvested power for different separation of magnets are presented.

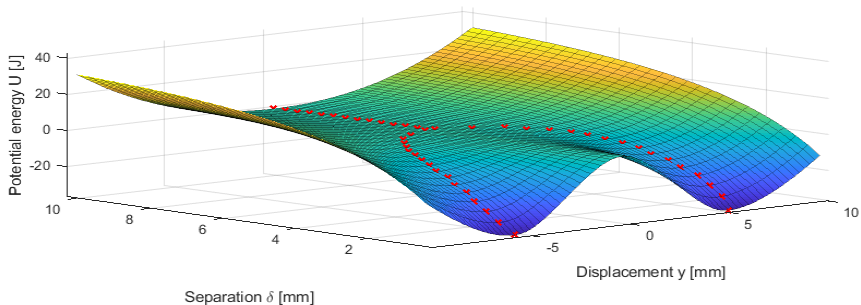
**Keywords:** bifurcation diagram, piezoelectric cantilever, magneto-elastic system, bistable, chaotic

## 1. Introduction

Stiffness nonlinearities are a hot topic in vibration energy harvesting technologies [1]. It could be a promising source of energy for wireless sensors and IoT applications. The presented system is analysed in form of a bimorph cantilever consisting of a fixed-free steel substrate with two piezoelectric patches [2]. The magneto-elastic interaction of two permanent magnets, one is a fixed tip mass and the other one is stationary, provides nonlinearity of the stiffness and a separation of both magnets is analysed.

## 2. Results and Discussion

Such a nonlinear system can behave in monostable or bistable regimes, depending on the separation gap between the magnets, see Fig. 1. Possible solutions include monostable oscillations, bistable in-well or cross-well oscillations, n-periodic oscillations, or chaotic behaviour.



**Fig. 1.** Potential energy of the nonlinear oscillator as a function of magnet separation.

The complex energy harvesting system is analysed as single-degree-of-freedom system and its behaviour is governed by following coupled differential equations:

$$m\ddot{y}(t) + b\dot{y}(t) + ky(t) + F_{mag}(y(t)) + \theta V(t) = D\ddot{z}\sin(\omega t)$$

$$\dot{V}(t) = \frac{1}{C_p} \left( \theta \dot{y}(t) - \frac{V(t)}{R} \right)$$

To study the influence of magnet separation distance on the system's dynamics the series of bifurcation diagrams were created, see Fig. 2. This bifurcation analysis is accompanied by attractor identification process, that analyses each time evolution based on Poincare points and colour-codes the solutions in said bifurcation diagrams. These diagrams show possible solutions, their periodicity, and generated power as well as a comparison to a linear system with no magnets. Many of these were created for various forcing conditions.

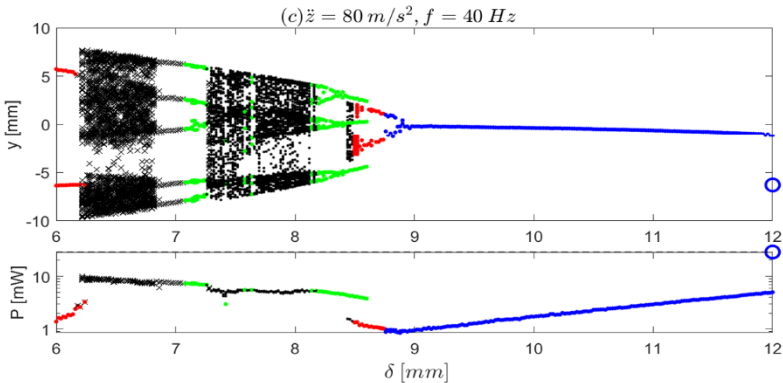


Fig. 2. Bifurcation diagram with magnet separation as a control parameter.

### 3. Conclusions

The behaviour and harvested power of the nonlinear coupled electromechanical system was analysed via bifurcation diagrams that differ in forcing conditions. The results can be used to find the magnet separation distance that generates the most energy for each pair of forcing conditions.

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### References

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