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Global Dynamics of a Polynomial Mechanical System

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Abstract: Using a bifurcational geometric approach, we study the global dynamics and solve the problem on the maximum number and distribution of limit cycles in a planar polynomial Euler-Lagrange-Liénard type mechanical system.

Keywords: Euler-Lagrange-Liénard system, bifurcation, field rotation parameter, singular point, limit cycle

1. Introduction

We consider an Euler-Lagrange-Liénard equation

$$\ddot{x} + h(x)\dot{x}^2 + f(x)\dot{x} + g(x) = 0 \tag{1}$$

and the corresponding dynamical system

$$\dot{x} = y, \dot{y} = -g(x) - f(x)y - h(x)y^2. \tag{2}$$

Particular cases of such a system were considered in [1-8]. There are many examples in the natural sciences and technology in which this and related systems are applied. Such systems are often used to model either mechanical or electrical, or biomedical systems, and in the literature, many systems are transformed into Liénard type to aid in the investigations. They can be used, e.g., in certain mechanical systems, where $f(x)$ represents a coefficient of the damping force and $g(x)$ represents the restoring force or stiffness, when modeling wind rock phenomena and surge in jet engines. Such systems can be also used to model resistor-inductor-capacitor circuits with non-linear circuit elements. The Liénard system has been shown to describe the operation of an optoelectronics circuit that uses a resonant tunnelling diode to drive a laser diode to make an optoelectronic voltage controlled oscillator [3].

There are also a number of examples of technical systems which are modelled with quadratic damping: a term in the second-order dynamics model, which is quadratic with respect to the velocity state variable. These examples include bearings, floating off-shore structures, vibration isolation and ship roll damping models. In robotics, quadratic damping appears in feed-forward control and in nonlinear impedance devices, such as variable impedance actuators. Variable impedance actuators are of particular interest for collaborative robotics [9, 10].

We suppose that system (2), where $g(x)$, $h(x)$ and $f(x)$ are arbitrary polynomials, has an anti-saddle (a node or a focus, or a center) at the origin [8].

2. Limit Cycle Bifurcations

Following [1], we study limit cycle bifurcations of (2) by means of canonical systems containing field rotation parameters of (2) [8].

Theorem 1. The Euler-Lagrange-Liénard polynomial system (2) with limit cycles can be reduced to one of the canonical forms:

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -x(1 + a_1 x + \ldots + a_{2l} x^{2l}) + \\ &+(\alpha_0 - \beta_1 - \ldots - \beta_{2k-1} + \beta_1 x + \alpha_2 x^2 + \ldots + \beta_{2k-1} x^{2k-1} + \alpha_{2k} x^{2k}) + \\ &+ y^2 (c_0 + c_1 x + \ldots + c_{2n} x^{2n}); \end{aligned} \tag{3}$$

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= x(x-1)(1 + b_1 x + \ldots + b_{2l-1} x^{2l-1}) + \\ &+(\alpha_0 - \beta_1 - \ldots - \beta_{2k-1} + \beta_1 x + \alpha_2 x^2 + \ldots + \beta_{2k-1} x^{2k-1} + \alpha_{2k} x^{2k}) + \\ &+ y^2 (c_0 + c_1 x + \ldots + c_{2n} x^{2n}), \end{aligned} \tag{4}$$

where α_0, α_2,…, α_{2k} are field rotation parameters, β_1, β_3,…, β_{2k-1} are semi-rotation parameters, and system (3) has the only singular point.

By means of the canonical systems (3) and (4), we prove the following theorem [8].

Theorem 2. The Euler-Lagrange-Liénard polynomial system (2) can have at most $k + l + 1$ limit cycles, $k + 1$ surrounding the origin and l surrounding one by one the other singularities of (2).

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