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# Global Dynamics of Thermomechanically Coupled Plates

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**Abstract:** Thermoelastic analysis of a shear deformable reduced model of laminated plates with von Kármán nonlinearities and cubic temperature along the thickness is presented. Parametric investigation of the response is accomplished by means of bifurcation diagrams, phase portraits and planar cross sections of the four-dimensional basins of attraction, in order to describe the local and global dynamical behavior of the model. Due to the thermomechanical coupling, the slow transient thermal dynamics is proved to be crucial in determining the steady mechanical response, which can be grasped only by proper developing an accurate global dynamics analysis in the multidimensional state space.

**Keywords:** Composite plates, Reduced order models, Thermomechanical coupling, Local and global dynamics

## 1. Introduction

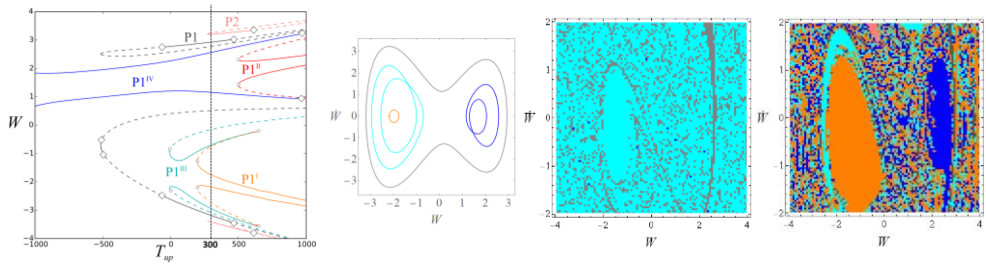
Thermomechanical coupling of materials and structures in a nonlinear dynamics environment represents a topic of great interest in fields like aerospace engineering, civil, and mechanical engineering, and in micro-electro-mechanics. To understand the basic, yet involved, effects of coupling on the finite amplitude vibrations of geometrically nonlinear structures, low-order models able to preserve the main features of the underlying continuum formulations turn out to be very important, as they get rid of the complicatedness generally occurring in the analysis and interpretation of nonlinear phenomena when using richer models [1,2]. Moreover, in the context of a global dynamics investigation, low-order models are crucial to perform the nonlinear analyses in a reduced state space, still with the possibility to obtain fundamental insight into thermal-structural interactions [3,4].

## 2. Results and Discussion

The thermomechanical plate model here used is derived within a unified modelling framework integrating mechanical and thermal aspects which, starting from the three-dimensional physics problem, moves to the two-dimensional and zero-dimensional formulations, as presented in [1]. Assumptions of third-order shear deformability and consistent cubic temperature variation along the thickness are imposed, and, in the absence of internal resonance between the plate transverse modes, a single-mode Galerkin approximation is adopted for the transverse displacement and the two independent bending and membrane temperatures. The choice of a dome-shaped prescribed temperature on the upper and lower surfaces allows to obtain the following three coupled nonlinear ODEs in terms of the deflection of the plate centre $W$, the membrane temperature $T_{R0}$, and the bending temperature $T_{R1}$ ($T_{up}$ and $T_{down}$ are the central values of the dome-shape temperature prescribed on the upper and lower external surfaces, respectively):

$$\ddot{W}+a_{12}\dot{W}+a_{13}W+a_{14}W^3+a_{15}T_{\mathrm{R1}}+a_{16}W\,T_{\mathrm{R0}}+a_{17}\cos(t)+a_{18}\left(T_{\mathrm{up}}+T_{\mathrm{down}}\right)W+a_{19}\left(T_{\mathrm{up}}\text{-}T_{\mathrm{down}}\right)=0$$
$$\dot{T}_{\mathrm{R0}}+a_{22}T_{\mathrm{R0}}+a_{23}\alpha_1\left(T_{\mathrm{up}}+T_{\mathrm{down}}\right)+a_{24}W\dot{W}+a_{25}e_0=0$$
$$\dot{T}_{\mathrm{R1}}+a_{32}T_{\mathrm{R1}}+a_{33}\dot{W}+a_{34}e_1+a_{35}\alpha_1\left(T_{\mathrm{up}}\text{-}T_{\mathrm{down}}\right)=0$$

Local and global nonlinear dynamics have been investigated through parametric analysis of the response by means of bifurcation diagrams, phase portraits and planar cross sections of the four-dimensional basins of attraction. The results highlight the non-trivial influence of the slow transient thermal dynamics on the steady outcome of the faster mechanical response, which can be unveiled only via a refined global analysis accomplished in the system actual multidimensional phase space. Indeed, local dynamics intrinsically neglects thermal transient, as continuation analyses are focused on the evolution of stationary responses. Conversely, global dynamics of the coupled system naturally considers both mechanical and thermal transient dynamics, thus representing the most suitable tool to comprehensively describe the response of the thermomechanical plate. When the thermal initial conditions are set to the relevant regime values, the basins of attraction of the coupled model display the same multistable response of the local dynamics analysis, which thus represents only a partial scenario (i.e. a particular section) of the overall four-dimensional plate behaviour.



**(a)** **(b)**

**Fig. 1.** At $T_{\mathrm{down}}$ = 100 K: **(a)** Local dynamics: bifurcation diagram and phase portraits of plate multistable periodic solutions; **(b)** Global dynamics: 2D sections of basins of attraction for null and steady state thermal initial conditions ($T_{\mathrm{up}}$ = 300 K).

## 3. Concluding Remarks

As a major result, the analyses have proved the ability of the coupled model to capture the actual behaviour of the physical system by correctly catching coupling effects, which turns out to be crucially important for all multiphysics systems, characterized by field variables evolving on different time scales. From a methodological viewpoint, this can be grasped only by complementing the local dynamics analysis with a deep investigation of the global features of the multidimensional response.

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