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Evaluation of the Reinjection Process in Type V Intermittency

SERGIO ELASKAR[1], EZEQUIEL DEL RÍO[2*], WALKIRIA SCHULZ[3]

1. IDIT and Dto. Aeronáutica, FCEFyN, Universidad Nacional de Córdoba and CONICET, Argentina [0000-0002-7250-0392]
2. Dto. de Física Aplicada, ETSIAE, Universidad Politécnica de Madrid, Spain [0000-0003-3384-9521]
3. Dto. Aeronáutica, FCEFyN, Universidad Nacional de Córdoba, Argentina [0000-0002-0015-6211]

* Presenting Author

Abstract: In this paper, a technique to evaluate the reinjection probability density function, and the probability density of the laminar lengths for type V intermittency is implemented. A family of maps with continuous and discontinuous RPD functions is studied. Several tests were performed, where the proposed technique was compared with the *M* function methodology, the classical theory of intermittency, and with numerical data. The analysis showed that the new technique captures accurately the random values of the numerical data. Therefore, the technique presented here is a useful tool to analytically calculate the statistical variables of type V intermittency.

Keywords: reinjection process, chaotic intermittency, transformation of random variables

1. Introduction

Maps that show intermittency have a local map and a non-linear map that produces the reinjection process. The intermittency type is determined by the local map, and the reinjection mechanism allows the return of the trajectories from the chaotic region to the laminar one. The reinjection probability density function (RPD) is used to quantify the reinjection process, and it expresses the probability of the trajectories to be reinjected in each point of the laminar zone [1]. Hence, the correct evaluation of the RPD function is fundamental to describe the chaotic intermittency phenomenon accurately. Type V intermittency occurs when a stable fixed point losses its stability through a collision with a non-differentiable point forming a channel between the map and the bisector line [2]. In this paper, we use the transformation of random variables to develop a methodology to analytically evaluate the reinjection probability density and the probability density of the laminar lengths (RPDL) for type V intermittency.

2. Model, Results and Discussion

We introduce the following family of maps

$$F(x) = \begin{cases} F_1(x) = \lambda_1 x + \varepsilon & \hat{x} \le x < 0, \\ F_2(x) = \varepsilon + x + \lambda_2 x^2 & 0 \le x < x_m, \\ F_3(x) = \hat{x} + \frac{(y_m - \hat{x})(y_m - x)^\gamma}{(y_m - x_m)^\gamma} & x_m \le x < y_m, \end{cases} \tag{1}$$

where $\hat{x}$ is the lower boundary of reinjection [1]. If we apply the transformation of random variables and we exclude the contributions that do not generate reinjection in the laminar zone [3], the RPD results

selaskar@unc.edu.ar

$$\phi(x) = \sum_{j\neq l}^{n} \left|\frac{dF_j^{-1}(x)}{dx}\right| \rho\left(F_j^{-1}(x)\right), \tag{2}$$

where l indicates the intervals that do not generate reinjection [3]. If the lower boundary of reinjection verifies $\hat{x} = x_0 - c$, where x_0 is the fixed point and c is the semi-amplitude of the laminar interval, we obtain continuous RPD functions as shown Figure 1 (centre). If the lower boundary of reinjection verifies $\hat{x} < x_0 - c$, type V intermittency shows discontinuous RPDs, which appear by two different processes of reinjection, one produced by $F_3(x)$ and the other by $F_1(x)$ as shown Figure 1 (left and right).

$$\phi(x) = \begin{cases} \phi_I(x) = \phi_1(x) + \phi_3(x) & x < F_1(x_0 - c), \\ \phi_{II}(x) = \phi_3(x) & x \geq F_1(x_0 - c). \end{cases} \tag{3}$$

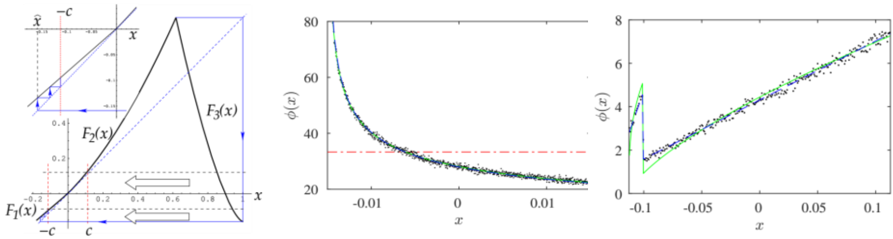

Fig. 1. Left: reinjection process. Centre: continuous RPD, $\gamma = 1.5$, $\varepsilon = 0.0001$, $c = 0.0015$, and the number of reinjected point is $N = 25 \times 10^5$. Right: discontinuous RPD, $\gamma = 0.5$, $\varepsilon = 0.001$, $c = 0.1128$, $N = 15 \times 10^5$ and $\hat{x} = -0.158449931412894$. Blue line: continuity technique. Green line: M function methodology. Red line: classical theory. Black points: numerical data.

3. Concluding Remarks

We presented a new technology to evaluate the statistical properties of type V intermittency, which was obtained from the transformation of random variables. We introduced and applied this technique to calculate continuous and discontinuous RPD and RPDL functions. We carried out comparisons of the theoretical results here obtained with those calculated by the M function methodology, the classical theory of intermittency, and numerical data. We have calculated the rate of convergence for all tests, and we have found that the process is convergent with rate of convergence $O(1/N^p)$ within $0.15 < p < 0.5$. Also, we found that the new technique works very accurately for different parameters, either with continuous or discontinuous RPDs. We conclude that the continuity technique has shown to have the ability to evaluate the reinjection probability density function and the probability density of the laminar lengths for type V intermittency.

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