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Strategies for amplitude control in a ring of self-excited oscillators

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Abstract: This paper presents different methods for amplitude control in a ring network of van der Pol oscillators. The slow flow equations for amplitude and phase difference are derived by the method of averaging. These slow flow equations are used to show that configuration symmetry of the network can be utilised to cause amplitude death. It is further shown that stable manifold associated with the equilibrium point at the origin can be used for control. Apart from these system characteristics, this paper also shows that a first order active control connected to any one of the oscillators can cause amplitude death in the whole network. The synchronising property of the ring network causes the propagation of control to all the oscillators for strong enough coupling. Frequency control to bring about synchronised dynamics and amplitude control to effect amplitude death are analysed in this work.

Keywords: Ring network, Synchronization, Amplitude death, Active control.

1. Introduction

Different methods like linear feedback and time-delay feedback have been proposed recently for control of coupled self-excited systems [1]. There have also been efforts to apply active control in forced van der Pol systems and to study its stability [2]. This work analyses different control strategies in a van der Pol ring network.

2. Van der Pol oscillators in a ring – Slowflow dynamics

N van der Pol oscillators connected in a ring network with linear dissipative coupling between nearest neighbours can be represented by [3]:

$$\ddot{x}_i + \varepsilon(x_i^2 - 1)\dot{x}_i + \omega_i^2 x_i = \sigma(\dot{x}_{i-1} - 2\dot{x}_i + \dot{x}_{i+1}) \tag{1}$$

Here, ϵ is the nonlinearity, σ is the coupling strength and ω_i is the linear natural frequency of each oscillator. The slow flow equations of Eq. (1) using the method of averaging [4] are in Eq. (2) and (3) where A_i is the amplitude and θ_i represents the phase difference.

$$\dot{A}_i = \frac{\varepsilon A_i}{2} - \frac{\varepsilon {A_i}^3}{8} + \frac{\sigma}{2}(A_{i-1}\cos\theta_{i-1} - 2A_i + A_{i+1}\cos\theta_i) \tag{2}$$

$$\dot{\theta}_i = (\omega_{i+1} - \omega_i) + \frac{\sigma}{2}\{\frac{A_{i-1}}{A_i}\sin\theta_{i-1} - \frac{{A_i}^2 + {A_{i+1}}^2}{A_i A_{i+1}}\sin\theta_i + \frac{A_{i+2}}{A_{i+1}}\sin\theta_{i+1}\} \tag{3}$$

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3. Results and Discussion

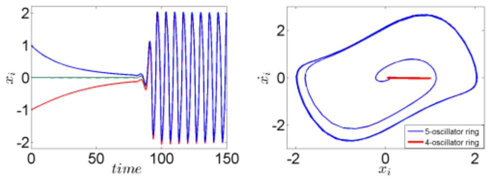

Fig. 1. (a) Frequency synchronization due to the presence of a 5th oscillator. (b) Limit cycle oscillation in a 5-oscillator ring changes to amplitude death in a 4- oscillator ring.

Configuration Symmetry: We extend the studies on influence of configuration symmetry on the dynamics of large ring networks [5] for framing control strategies. Two rings of N = 5 and N = 4 are considered with oscillator parameters as $\epsilon_i = 1$, $\sigma_i = 0.1$, $\omega_i = 1$. The slow flow equations (Eq. (2), (3)) are numerically integrated by Runge-Kutta method. For the 4-oscillator ring, fixed point at the origin has a stable manifold given by $S = \{(a,0), (-a,0), (a,0), (-a,0) \ldots\}$, a being any real positive number. This stable manifold does not exist in a 5-oscillator ring, as shown in Fig. 1(a). This amplitude control through frequency synchronization is destroyed by the removal of an oscillator which makes the ring symmetric. Under the same set of oscillator parameters, we observe the annihilation of amplitude across the entire network (Fig. 1(b)) in a 4-number ring.

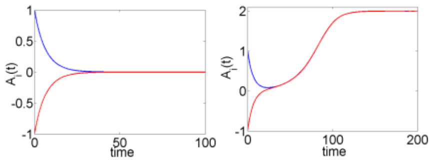

Fig. 2. Evolution of amplitude in a symmetrical ring. (a) Amplitude annihilation (b) Amplitude preservation

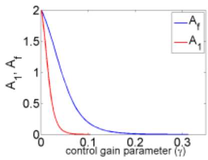

Fig. 3. Spread of amplitude death across the ring by active control.

Perturbation in stable manifolds: In even number rings, the symmetry in the network aids the annihilation of amplitude. Beginning the dynamics on stable manifold given by $S = \{(a,0), (-a,0), (a,0), (-a,0) \ldots\}$ leads to amplitude death as shown in Fig. 2(a). Any slight perturbation $(a+\Delta a)$ leads to restoration of the amplitude and achievement of synchronisation as given in Fig. 2(b).

Influence of an active control: Amplitude control in the whole network can also be realised by connecting a first order active control to any one oscillator in the ring. The synchronisation property of the network helps in the spread of amplitude control in one oscillator to the entire network. Fig. 3 shows amplitude death in first oscillator (which is connected with active control) and the oscillator farthest from it.

4. Concluding Remarks

This paper investigated three methods for amplitude annihilation or restoration in a ring of limit cycle oscillators. Whereas the first two methods utilised inherent properties of the system, the third method utilises an external control mechanism. But the propagation of the control throughout the network happens due to its synchronising property.

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