

## Optimal Control of Resonance Radiation Processes in Laser Isotopes Separation Systems and Devices

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**Abstract:** The paper is devoted to presentation of an effective numerical approach to construction of an optimal laser photoionization (LPI) isotope separation scheme (technology). The construction of the LPI optimal models requires quantum numerical modelling of the corresponding radiation processes (due to collisions, or ionization by a pulsed electric field etc) in complex atomic systems. The optimal laser action model and density matrices formalism are used for numeric computing the optimal LPI scheme parameters. One of the possible LPI optimization model entails a search for an optimal shape of a laser resonant pulse in order to obtain maximum number of ionized particles in the LPI scheme with ionization at the final stage by a pulsed electric field. The solution of the optimal control problem gives the condition for the existence of optimal laser exposure in the form of a single pulse and its optimal shape.

**Keywords:** optimal control, laser isotopes separation scheme

### 1. Introduction. Optimal Control Task for Resonance Radiation Processes

A multi-staged LPI isotope separation method comprises a selective laser pulse excitation of the isotope atoms into the excited and further Rydberg states (first and middle stages) and their ionization in the final stage [1-3]. The criteria to be considered on pursuing optimality and efficiency of multi-staged LPI scheme include the minimum required energy densities of laser pulses, a maximum yield of ionization and its selectivity. A construction of optimal isotope separation LPI models requires numerical modelling of the corresponding radiation processes in atomic ensemble. One of the possible models for LPI optimization entails the search for a laser pulse optimal shape in order to obtain maximum number of ionized particles in the LPI scheme with ionization at the final stage [1,2]. If the quantity  $R$  is an atomic ionization rate,  $x_2$  is the normalized population of the excited states of the atom and  $\bar{\tau}$  is the duration of radiation pulse, then the corresponding performance criterion is reduced to the minimization of the functional ( $J$ ), which determines the number of ionized atoms:

$$J = - \int_0^{\bar{\tau}} R(\tau) x_2 d\tau \rightarrow \min; \quad (1)$$

A simple ionization model can be represented by a system of standard Bloch's-like equations with the following initial conditions [3]:

$$dx_1/d\tau = x_2 - \bar{u}(x_1 - x_2), x_1(0) = 1; \quad (2)$$

$$dx_2/d\tau = -[R(\tau) + 1]x_2 + \bar{u}(x_1 - x_2), x_2(0) = 0; \quad (3)$$

$$dx_3/d\tau = \tilde{u}, x_3(0) = 0, x_3(\tau_f) = E_f; \quad (4)$$

where  $0 \leq \tau \leq \tau_f, \tilde{u}(\tau) \geq 0$ ;  $x_1$  is the normalized population of the ground state of an atom;  $\tilde{u} = u/\gamma$  is a dimensionless rate of induced emission and absorption processes of the resonant radiation;  $\gamma$  is the probability of spontaneous decay per unit time;  $\tau = t\gamma$  is a dimensionless time;  $R=R'/\gamma$  is a dimensionless ionization rate from excited state;  $u(t)=\sigma_{12}I_1(t)/\hbar\omega_{21}$  is the rate of induced transitions ( $1 \rightarrow 2$  transition);  $\omega_{21}$  is a radiation frequency;  $\sigma_{12}$  is the absorption cross section;  $R'(t) = \sigma_{ph}I_2(t)/\hbar\omega_{ph}$  is the photoionization rate;  $\omega_{ph}$  is a radiation frequency for photoionization;  $\sigma_{ph}$  is a photoionization cross section;  $I_1, I_2$  are the intensities of laser pulses for excitation from the ground state and ionization from the excited one;  $E_f, \tau_f$  are an energy and duration of pulse.

## 2. Results and Discussion

The solution of the optimal control problem gives the condition for the existence of optimal laser exposure in the form of a single pulse as follows:

$$p = \exp(-2E_f)\{1 + 2/R_0 + \exp[-(R_0 + 1)\tau_f]\} / (1 + 2/R_0)\{1 - \exp[-(R_0 + 1)\tau_f]\}, p \geq 1 \quad (6)$$

where the parameter  $p$  has an expression similar to the analogous parameter in the Krasnov-Shaparev-Shkedov scheme [2]. The optimal resonant laser pulse is given by:

$$\tilde{u}(\tau) = \begin{pmatrix} E_1 \delta(\tau) + \tilde{u}'(\tau), \tau \in [0, \tau_1] | p < 1 \\ 0, \dots, \dots, \tau \in [\tau_1, \tau_f] | p < 1 \\ E_f \delta(\tau), \dots, \dots, \tau \in [0, \tau_f] | p \geq 1 \end{pmatrix} \quad (7)$$

where  $\delta(\tau)$  is the Dirac delta-function,  $E_l$  ( $E_l < E_f$ ) is the amplitude of the  $\delta$ -pulse;  $\tau_m$  is the time parameter, which is matched numerically ( $\tau_m < \tau_f$ );  $R_0$  is the ionization rate from the ground state of atom. More mathematical and physical details of the model can be found in [2-4].

## 3. Concluding Remarks

An effective numerical approach to construction of an optimal laser photoionization LPI isotope separation scheme for radioactive elements is presented. It is numerically solved a LPI optimization problem, connected with a search for an optimal shape of a laser pulse of resonant radiation in order to obtain maximum number of ionized particles in the LPI scheme.

## References (10 point, bold)

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