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Reference model trajectory tracking in continuous-time sliding mode control

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Abstract: Controller design for a continuous-time system subject to nonlinear disturbance is a complex task with many challenges. One must ensure that the effects of disturbance on system dynamics are properly compensated, while at the same time keeping state and input constraints in mind. Disturbance rejection by itself is easily achieved using sliding mode controllers (SMC). However, such controllers give no insight into the dynamics of individual state variables, which may be subject to physical constraints. In this paper we propose a solution, which allows one to benefit from the disturbance rejection property of SMC and at the same time obtain better insight into system dynamics. The proposed approach involves a reference model obtained from a canonical form of the controlled system. A sliding mode control strategy is applied to the plant with the aim of driving its state alongside that of the model. Then, since model dynamics are inherently simpler, one can modify them to impose specific constraints on the motion of the system. It is demonstrated that, when the proposed SMC strategy is applied, individual states of the original plant always exactly follow those of the model, regardless of uncertainties.

Keywords: robust control, sliding mode control, trajectory tracking, continuous-time systems

1. Introduction

Continuous-time sliding modes are an excellent tool for controlling system subject to nonlinear uncertainties, as they are able to reject the effect of matched disturbance on the system entirely [1]. However, since sliding mode controllers are based on the virtual system output referred to as a sliding variable, they do not typically provide any direct way of controlling the magnitude of individual system states. This is a concern, since in many practical applications, these states may need to follow strict physical constraints. Motivated by this problem, in our paper we propose a trajectory tracking sliding mode control scheme which imposes strict bounds on individual system states, and at the same time ensures the valuable disturbance rejection property that sliding mode controllers are known for.

In particular, in our research we explore sliding mode control of continuous-time dynamical systems transformed to the following canonical form

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t) + \boldsymbol{d}(t), \tag{1}$$

where state matrix $\boldsymbol{A}$, input distribution vector $\boldsymbol{b}$ and disturbance $\boldsymbol{d}$ are expressed as

$$\boldsymbol{A}=\begin{bmatrix}0 & 0 & 1 & \cdots & 0\\ 0 & 0 & 0 & & 0\\ \vdots & & & \ddots & \vdots\\ 0 & 0 & 0 & & 1\\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1}\end{bmatrix},\quad \boldsymbol{b}=\begin{bmatrix}0\\0\\ \vdots\\0\\1\end{bmatrix},\quad \boldsymbol{d}(t)=\begin{bmatrix}0\\0\\ \vdots\\0\\f(t)\end{bmatrix} \tag{2}$$

for a certain nonlinear function $f(t)$ and constants $a_0, .., a_{n-1}$. The objective of this paper is to design a reference model of system (1) which will ensure desirable state dynamics in the absence of disturbance. Then, a trajectory tracking control scheme will be applied to the original plant with the aim of driving its state towards that of the model, thus satisfying state constraints imposed on the system.

2. Results and Discussion

The approach proposed in our paper uses a disturbance-free reference model of plant (1), in which feedback gains $a_0, .., a_{n-1}$ are omitted. In other words, the model is essentially an n-th order integrator, which is inherently easy to control. In particular, we use linear state-feedback control with feedback gains selected to ensure that all states of the model stay within specific bounds. Then, a sliding mode control strategy is applied to the plant in order to drive its states alongside those of the model.

It is demonstrated that the proposed continuous-time sliding mode control scheme with a reference model ensures that the specified state trajectories are always followed exactly. This is achieved even in the presence of nonlinear uncertainties, which means that physical constraints placed on the system can be satisfied even in difficult conditions. Similar model reference schemes have been proposed in the past [2], but in this work we explicitly prove that exact trajectory tracking can be achieved in an uncertain continuous-time system.

It is known that perfect disturbance rejection in sliding mode control is only possible in a purely continuous-time system. When the digital implementation of the control scheme is considered, one observes the undesirable chattering phenomenon, i.e. high frequency oscillations around the target state [3]. This phenomenon is also present when our approach is applied to the plant. However, it can be demonstrated that, if proper disturbance compensation based on past data is applied, the magnitude of these oscillations can be reduced to a negligible level [4]. Thus, it can be concluded that for all practical purposes, our approach ensures full disturbance rejection and ensures boundedness of all system states.

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