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Sliding of tabouret with elastic legs on a rough surface under the action of a small lateral force.

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Abstract: The features of the behavior of a dynamically asymmetric body with two elastic supports on a plane with dry friction are discussed. A constant lateral force is applied to the body. The equations of motion are the dynamical system of variable structure. In a position of seemingly obvious vertical equilibrium, the equilibrium conditions are not met. A motion is described, during which one or both supports slide depending on the value of the coefficient of friction. The presence of elasticity in the supports leads to permanent sliding of the body even in the case of a relatively small lateral force.

Keywords: elastic force, dry friction, dynamical system of variable structure.

1. Introduction

When modelling the braking of a vibrating robot on two supports on a rough plane, problems were found with the description of the friction force [1]. Similar problems were encountered in modelling other problems of mechanics [2, 3]. To regularize the description of the interaction of the body with the support, the compliance of the supports is introduced into the model.

2. Description of the mechanical system behavior

Let a heavy body *ABCD* ($AB = 2a$, $AD = 2b$) of mass *m* (Fig. 1) perform plane-parallel motion, resting on a horizontal rough plane with two elastic supports AA_1 and BB_1. When the supports are not stressed, their length $AA_1 = BB_1 = l_0$. The centre of mass G_1 of the body is displaced from its centre *G* by a distance *d* along the straight line *DC*. The springs of the supports act on the body by forces $F_{el1} = -k(l_1 - l_0) - h\dot{l}_1, F_{el2} = -k(l_2 - l_0) - h\dot{l}_2$, where *k* is the stiffness coefficient, *h* is the damping coefficient of the springs.

The following external forces act on the system: gravity $\mathbf{P} = m\mathbf{g}$ ($\mathbf{g}$ is the acceleration of gravity), normal $\mathbf{N_1}, \mathbf{N_2}$ and tangential $\mathbf{F_{fr1}}, \mathbf{F_{fr2}}$ reactions of the supports, which, in the case of sliding of the supporting legs, are related to each other by the Coulomb law: $F_{fr1} = -\mu signum(V_{x_{A1}})\left|N_1\right|$, $F_{fr2} = -\mu signum(V_{x_{B1}})\left|N_2\right|$, as well as the lateral force $\mathbf{F_e}$ applied at the center of mass G_1. Note that both legs slide if $\mu = 0$.

As generalized coordinates, we choose the coordinates x_{G_1}, y_{G_1} of the center of mass G_1 and the angle φ between the vertical and the direction of the elastic supports. We study the possible types of system behavior. The dynamical system under consideration is the system of variable structure. In

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general, when both legs slide, the mechanical system has 3 degrees of freedom. The equations of motion in the case of sliding of both legs can be represented as follows:

$$m\ddot{x}_{G_1} = F_{fr1} + F_{fr2} + F_e$$

$$m\ddot{y}_{G_1} = -mg + N_1 + N_2$$

$$J\ddot{\varphi} = N_1((l_1 + b)\sin\varphi + (a + d)\cos\varphi) - N_2(-(l_2 + b)\sin\varphi + (a - d)\cos\varphi) - (F_{fr1} + F_{fr2})(l_0 + b + y_{G_1})$$

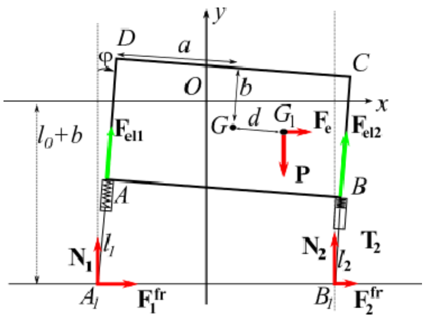

Fig. 1. The body on a rough plane.

During periods of time when one of the supports does not slide, the system has two degrees of freedom. This is equivalent to constraining $x_{A_1} = const$ or $x_{B_1} = const$. In this case, the dynamical system consists of two equations, for example, for generalized coordinates y_{G_1}, φ. In this case, the second support stops only at those moments of time when the angular velocity turns to zero.

A numerical study of the obtained dynamical system of variable structure for various values of the friction coefficient was carried out.

3. Conclusion

It is shown that even the simplest transient motion of the body from a certain state of rest to an equilibrium position is accompanied by slipping of one or both supports, depending on the value of the friction coefficient. With friction equal to the applied lateral force, at which the body on rigid supports would not budge, the centre of mass of the system gains a nonzero velocity.

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