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Application of the differential transform method to the study of the Duffing system with fractional damping and stiffness.

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Abstract: Non-linear differential equations with fractional derivatives provide a satisfactory description of many real dynamical systems and life phenomena. In this study, a relatively new method for solving the fractional differential equations, i.e. the differential transform method (DTM), is applied to test the Duffing system with fractional stiffness and damping.

Keywords: Duffing system, fractional equations, differential transform method

1. Introduction

The DTM was proposed by Zhou [1] to solve both linear and non-linear problems in electrical circuits. The differential transform of a function *f(x)* is defined in a similar way to the Taylor series expansion:

$$F(k)=\frac{1}{k!}\frac{d^k f(x)}{dx^k}\bigg|_{x=0} \tag{1}$$

and the inverse transform is defined as

$$f(x)=\sum_{k=0}^{\infty} F(k)x^k. \tag{2}$$

The method was extended to solve fractional differential equations by Arikoglu and Ozkol [2], with the inverse transform described by the equation

$$f(x)=\sum_{k=0}^{\infty} F(k)(x-x_0)^{k\alpha}, \tag{3}$$

where α is the order of fraction. Despite the fact that DTM derives from the Taylor series expansion, it does not require that the derivative be evaluated symbolically. Instead, relative derivatives are calculated iteratively. Differential transforms of the various expressions are given with proofs in a number of studies (for example [3]). Due to the method's simple iterative scheme, solving fractional differential equations by DTM is numerically fast and gives very accurate solutions.

2. Results and Discussion

In this study, we analyse the Duffing oscillator system with additional damping and stiffness factors that depend on previous states of the system and can, therefore, be described by fractional derivatives. In preliminary tests, the method was approved by comparing the Duffing System solutions obtained useing DTM and Runge-Kutta 4 algorithms. An example of typical results is presented in Fig. 1.

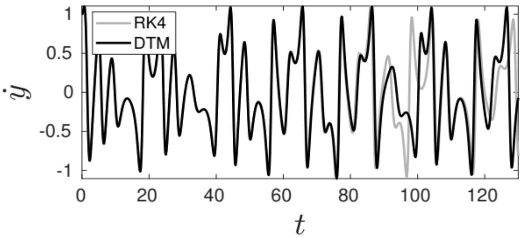

Fig. 1. Solutions of the integer Duffing system, obtained using DTM and RK4 methods.

The calculations assumed an integration step of h=0.01 s (DTM) and h=0.001 s (RK4). Both solutions change in time identically (by eye) for about 80 seconds. A comparison of the DTM and ode45 Matlab function (RK method with a variable time step) shows that it is possible to obtain the convergence of both solutions for hundreds of seconds provided that the integration step in the ode45 function is appropriately limited.

The analysis starts with the following equation:

$$\ddot{x} = -b\dot{x} - b_p D^{\beta_p}\dot{x} - b_m D^{\beta_m}\dot{x} + k_p D^{\alpha_p} x + kx - cx^3 + f_0 \sin(\omega t), \tag{4}$$

where b_p, b_m, an k_p are the coefficients of the fractional factors with the fractional orders of β_p, β_m, and α_p, respectively. These coefficients define the fractional forces that disturb the Duffing system, whereas the fractional orders determine the influence of the history of the system on the fractional terms. This paper presents the results of the fractional system simulations performed for different values of the coefficients and orders of fractional factors.

3. Concluding Remarks

In each simulation, the average kinetic and potential energies of the system are calculated for the selected time period. As a result, it is easier to distinguish vibration modes in the comparison diagrams and thus assess the influence of fractional factors on the vibration energy accumulated in the system. The results of this study can be useful for designing energy harvesting or dissipation systems using fractional two-well oscillators.

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References

[1] ZHOU JK: Differential Transformation and Its Application for Electrical Circuits. *Huazhong University Press*, Wuhan, 1986 (in Chinese).

[2] ARIKOGLU A, OZKOL I: Solution of fractional differential equations by using differential transform method. *Chaos, Solitons & Fractals 2007, **34**:1473-1481.*

[3] ODIBAT Z, MOMANI S, ERTURK VS: Generalized differential transform method: Application to differential equations of fractional order. *Capplied Mathematics and Computation 2008, **197**:467-477.*