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# Uncertainty evaluation by the bootstrap for the staircase fatigue limit test data

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**Abstract:** The uncertainty analysis is carried out to evaluate the staircase test for determining the fatigue limit of a material. Using the bootstrap method on the staircase data to obtain the scatter of mean and standard deviation of the distribution for the fatigue limit. Combined with the results of a bending fatigue test, the evaluation process and results of the uncertainty of the staircase data under several strain levels are given. The outcomes of uncertainty based on the experimental application prove the feasibility of the bootstrap methods for staircase data.

**Keywords:** fatigue limit, staircase method, uncertainty, bootstrap, resampling

## 1. Introduction

The fatigue test of metallic materials is a commonly used way to evaluate the mechanical properties and the reliability of materials. Staircase test method [1] is widely practiced to determine the fatigue limit of material. Though the staircase test provides a precise value of fatigue limit, it is hard to measure the uncertainty from test results due to the test benches, sensors and specimens. To investigate this problem, an increasing number of recent researches on post-processing [2] and uncertainty evaluation [3] [4] of the staircase method can be observed in literature. In this paper, a new methodology is proposed to assess the uncertainty on fatigue limit involved in the staircase test.

## 2. Results and Discussion

By convention, no more than 30 specimens [4] are loaded in real staircase experimental test. Bootstrap method provides an efficient way to reproduce more samples from small-size dataset. It utilizes multiple random draws from experimental test results to make statistical inferences about the primary population. To examine the uncertainty in experiment, we propose:

**Step 1**: Representing the dataset from staircase method by $X=[x_1,x_2,\cdots,x_n]$, and dividing them to different part according to different strain (or stress) level *j*, $X^j=[x_1^j,x_2^j,\cdots,x_m^j]$, that is $X^1 \cup X^2 \cup X^j = X$;

**Step 2**: Start sampling with random stress level *j* form the test data. If the sampled specimen is failure, the next sample is selected form specimens in lower stress level *j*-1; otherwise, the next sample selected form specimens in upper stress level *j*+1;

**Step 3**: Repeating the step 2 until all collected specimens can accurately represents the true distribution of fatigue limit, which should be more than 50[1] as one group data; each group is still can be seen as a virtual staircase test data, donated as $\bar{X}_1=[\bar{x}_1,\bar{x}_1,\cdots,\bar{x}_M]$;

**Step 4**: Repeating step 2 and step 3, to sample enough groups data, $[\bar{X}_1, \bar{X}_2, \cdots, \bar{X}_N]$, to estimate the uncertainty;

**Step 5**: Fitting the distribution of each group data and obtain mean and standard deviation, and then fitting the distribution of all means and standard deviations as uncertainty estimation.

To validate the proposed method, an experimental fatigue limit test is performed on low carbon steel DC01 resulting in a dataset of 36 specimens. A close looped strain-control on electrodynamic system is carried out with a maintain of the excitation until 1e6 cycles. The staircase result is post-processed by the Kernel Density Estimation (KDE) that reveals the mean and standard deviation of fatigue limit are 1366.88$\mu m/m$ and 24.36$\mu m/m$, respectively. To apply the bootstrap method, 30 groups are drawn from the dataset each contains 50 samples. The related mean and standard deviation distributions are depicted in Fig. 1.

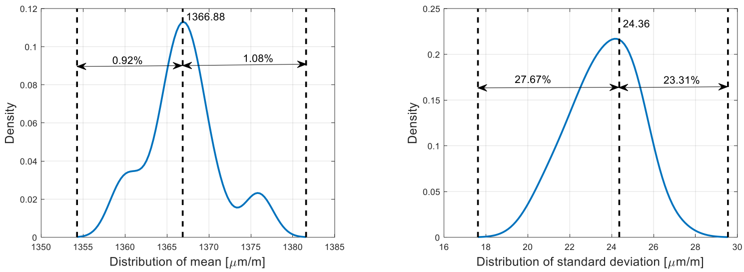


**Fig. 1.** Distributions of mean and standard deviation

From the results in Fig. 1, small interval in the mean value of fatigue limit is observed, while huge interval takes place in standard deviation. The staircase test offers a good estimation of median value but with higher uncertainty to standard deviation estimation.

## 3. Concluding Remarks

Evaluation on statistical uncertainty in the staircase test for fatigue limit based on bootstrap methods is presented. To avoid repetition of the staircase tests, a sampling with replacement procedure for staircase data is used to quantify the uncertainties of the mean and standard deviation of the fatigue limit obtained from one staircase test. The results show that the standard deviation of fatigue limit is more dispersive with more than 20%, leading to conclude that it should be conservative to use staircase results in fatigue design.

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