

POWER BOUNDED AND UNIFORMLY MEAN ERGODIC COMPOSITION OPERATORS ON LARGE CLASS OF FOCK SPACES

WERKAFERAHU SEYOU^{*1}, TESFA MENGESTIE², AND JOSÉ BONET³

1. Mathematics Department, Kotebe Metropolitan University
E-mail address: Werkaferahus@gmail.com. [<https://orcid.org/0000-0002-6035-9440>]
 2. Mathematics Section, Western Norway University of Applied Sciences, Klin-genbergvegen 8, N-5414 Stord, Norway. E-mail address: Tesfa.Mengestie@hvl.no.
 3. Universitario de Matematica Pura y Aplicada, Universitat Polit_ecnica de Valencia, Spain
E-mail address: jbonet@mat.upv.es
- * Presenting Author

Abstract: For a given holomorphic mapping ψ and f on a complex plane C , composition operator induced by ψ is defined by $C_\psi f = f \circ \psi$. Composition operator C_ψ acting on Fock spaces $F_\phi^p, 1 \leq p < \infty$ generated by weights ϕ satisfying $\limsup_{|z| \rightarrow \infty} \phi(z) = \infty$ is bounded if and only if $\psi(z) = az + b, |a| \leq 1$ and $b = 0$ whenever $|a| = 1$. We have shown that every bounded composition operator acting on the Fock spaces $F_\phi^p, 1 \leq p < \infty$ is power bounded. Mean ergodic and uniformly mean ergodic composition operators on the spaces are also characterized.

Keywords: Composition operators, Fock spaces, power bounded operator, mean ergodic operator, uniformly mean ergodic operator.

1. Introduction

Given an entire function ψ on the complex plane C , the composition operator induced by ψ is defined by $C_\psi f = f \circ \psi$ for each entire function f . The study of composition operators acting between spaces of analytic functions defined on the disc or on the complex plane has quite a long and rich history. Many properties of composition operators on spaces of entire functions have been also investigated. In the frame of Fock spaces, Carswell, MacCluer and Schuster [1], Guo and Izuchi [2], and first two authors [4, 3] characterized bounded and compact composition operators on the Fock spaces $F_\phi^p, 1 \leq p < \infty$. They showed that only the class of linear $\psi(z) = az + b, |a| \leq 1$ and $b = 0$ whenever $|a| = 1$ induces bounded composition operators, when $\limsup_{|z| \rightarrow \infty} \phi(z) = \infty$. Compactness of the composition operator was described by the strict requirement $|a| < 1$.

We recall some definitions. Let $\phi: [0, \infty) \rightarrow [0, \infty)$ be a twice continuously differentiable function. We extend ϕ to the whole complex plane by setting $\phi(z) = \phi(|z|)$.

The generalized Fock spaces F_φ^p , $1 \leq p < \infty$ associated with φ are the spaces consisting of all entire functions f such that

$$\|f\|_{(\varphi,p)} = \left(\int_C |f(z)|^p e^{-p\varphi(z)} dA(z) \right)^{1/p} < \infty,$$

Where dA denotes the usual Lebesgue area measure on the space of complex plane C .

Let X be a Banach space, and T a bounded operator acting on X . We denote the n -th ergodic mean T_n by

$$T_n := \sum_{m=1}^n T^m.$$

T is said to be power bounded if $\sup_{n \in \mathbb{N}} \|T^n\| < \infty$. We say that T is mean ergodic if there exists

a bounded operator P on X such that for every f in X ,

$$\lim_{n \rightarrow \infty} \|T_n f - P f\| = 0,$$

and uniformly mean ergodic if the above point-wise convergence is uniform.

In this paper we have characterized some dynamical properties of composition operators: power bounded, mean ergodic, and uniformly mean ergodic composition operators on Fock spaces when $\limsup_{|z| \rightarrow \infty} \varphi(z) = \infty$.

2. Results

We state the main result of our paper as follows.

Theorem 2.1. Let C_ψ be a bounded composition operator on Fock space F_φ^p , $1 \leq p < \infty$ with $\limsup_{|z| \rightarrow \infty} \varphi(z) = \infty$. Then

- (1) C_ψ is power bounded.
- (2) If $\psi(z) = az + b$: $|a| < 1$, then C_ψ is uniformly mean ergodic.
- (3) If $\psi(z) = az + b$: $|a| = 1$ and a is a root of the unity, then C_ψ is uniformly mean ergodic.
- (4) If $\psi(z) = az + b$: $|a| = 1$ and a is not a root of the unity, then C_ψ is mean ergodic but not uniformly mean ergodic.

References

- [1] B. J. Carswell, B. D. MacCluer, A. Schuster, Composition operators on the Fock space. Acta Sci. Math. (Szeged), 69, 871-887 (2003).
- [2] K. Guo, K. Izuchi, composition operators on Fock type space, Acta Sci. Math. (Szeged), 74, 807-828 (2008).
- [3] T. Mengestie, W. Seyoum, Spectral Properties of Composition Operators on Fock-Type Spaces, Quaestiones Math. <https://doi.org/10.2989/16073606.2019.1692092> (2019)
- [4] T. Mengestie, W. Seyoum, Topological and dynamical properties of composition operators, Complex Analysis and Operator Theory <https://doi.org/10.1007/s11785-019-00961-8> (2020)
- [5] W. Seyoum, T. Mengestie, J. Bonet, Mean ergodic composition operators on generalized Fock space, RACSAM, <http://doi.org/10.1007/s13398-019-00738-w> (2019)