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Bifurcations in PieceWise-Smooth Systems Associated to Migration

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Abstract: Human migration is a worldwide phenomenon. One of the multiple ways of temporal or periodic migration is tourism. Even though tourism is a relatively new concept in human history, its consequences are noticeable, both on a local and global scale. In general, the introduction of tourism in one country, or community, has not been planned appropriately, or simply not planned at all. Overexploitation of natural resources and inadequate behaviors leads to a conflict between tourists and locals that makes the tourist economy not sustainable. The fall of tourism in 2019-2020 due to a pandemic episode has shown how dependent the economy could be on the tourism industry. In the new start, politicians, social actors, and stakeholders in general, need a tool that gives them the opportunity to plan actions and policies. The modeling of tourist flows could be the tool to face the issue, looking for the balance between natural and socioeconomic resources, the interests and rights of tourists and locals. This paper proposes and analyses a mathematical model of nonlinear differential equations, which allows studying the dynamic interaction between tourists and residents. We use analytical and numerical methods to solve the proposed Filippov piecewise-smooth system. Invariance, equilibrium points, bifurcations, and switching surface have been studied.

Keywords: tourism, migration, bifurcation, PWS systems, non-linearity.

1. Introduction

The Filippov system, which relates tourists (T) to residents (R) in a city, is given by the following piecewise smooth system.

$$\begin{cases} \dot{R} &= \alpha_3 R \left(1 - \dfrac{R}{k-k_1}\right)\left(\dfrac{R}{k_2} - 1\right) - \alpha_4 R T \\ \dot{T} &= \alpha_1 T \left(1 - \dfrac{T}{k_1}\right) - \alpha_2 R\, g(R,T). \end{cases}$$

$$g(R,T) = \begin{cases} 1 & \text{si} \quad h(R,T) > 0 \\ 0 & \text{si} \quad h(R,T) < 0 \end{cases} = \begin{cases} 1 & \text{si} \quad T > T_0 \\ 0 & \text{si} \quad T < T_0 \end{cases}$$

where $\alpha 1$ and $\alpha 3$ are natural growth factors, $\alpha 2$ and $\alpha 4$ are factors related to tourist actions and policies, k's are asymptotic population limits, and h is the Heaviside function [1,2].

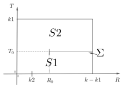

Fig. 1. State space for the Filippov system

Dynamics are defined by different systems of differential equations in regions S1 and S2, being Σ the switching surface (Fig.1). On this surface, the set of crossing points are represented by a dashed curve. In the dashed curve, the projection of the dynamics in the normal surface direction of the two sub-systems are non-zero and have the same sign. The orbits of the Filippov system cross the surface, i.e. the orbit reaching a point x on the surface from Si, concatenates with the orbit entering Sj ($j \neq i$) from x. The solid curve corresponds to sliding points. The projections of the dynamics on the solid part of the switching surface are non-zero and have different sign. In this case this set is a stable segment. Orbits reaching one of these points, from either region, slide on the surface.

2. Results and Discussion

2.1 Invariance.

Invariance in the state space (R,T) has been analyzed. R and T axes are invariant, and the other lines which determine the rectangle are not crossed by the orbits. This allows us to ensure that for $\alpha 2 > 0$ and $\alpha 4 > 0$ an orbit that begins at an interior point, remains in the rectangle determined by the coordinated axes and the lines R = k - k1 and T = k1.

2.2 Equilibrium points.

In S1 we have found 3 unstable equilibrium points (named E1, E2 and E3) on the R axis. There are saddle and sources, but none of them corresponds to attractors, which would mean the extinction of the tourists or tourist and resident populations. In S2 we have found an equilibrium point E4 located on the T axis and the equilibrium points resulting from the possible intersection of two curves. The existence from one to five equilibrium points in S2 depends on the T_0 value. The stability of the equilibrium points has been analysed using MatLab. E4 is a stable point. This case would correspond to the disappearance of the residents and the total occupation of the city by the tourists. On the switching surface, sliding points and pseudo-equilibrium points [3] deserves special attention.

3. Concluding Remarks

The relationship between tourist and resident populations has been modeled using a piecewise smooth Filippov system. The equations, for a determined set of parameters, have been solved analytically and numerically (using MatLab). Invariance, equilibrium points, bifurcations, and switching surface have been studied. The obtained results can be associated with situations of stability decrease, increase, or disappearance of tourist and resident populations.

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