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Dynamics of the Chaplygin sphere on a moving plane

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Abstract: This paper addresses the problem of a dynamically asymmetric balanced sphere rolling on a plane performing horizontal periodic oscillations. It is shown that, in the absence of gyrostatic momentum, the system admits additional integrals of motion. It is shown that the system under consideration has no steady-state solutions which exist in the unperturbed problem (in the absence of the plane's oscillations). The rolling of the sphere along a straight line can occur only in the direction of the plane's oscillations. In this case, the angular velocity vector of the sphere lies in the horizontal plane and is perpendicular to the direction of the plane's oscillations. Special attention is given to the problem of the controlled motion of the sphere by means of the variable gyrostatic momentum. Algorithms for controlling the motion of the sphere are constructed to make it rotate about the vertical and to make it move in a straight line with constant velocity relative to the moving and absolute coordinate systems.

Keywords: Chaplygin sphere, vibrating plane, nonholonomic constraint, permanent rotations, control

1. Introduction

The problem of the rolling of a dynamically asymmetric balanced sphere (the Chaplygin sphere) is one of the classical problems of nonholonomic mechanics, which was first posed and investigated in the work of S.A. Chaplygin [1]. Chaplygin obtained equations of motion of the sphere and integrated them by quadratures. Later, further inquiry into this problem was carried out in [2-4], in particular, a bifurcation analysis was performed, the trajectories of the sphere in absolute space were classified etc. This paper deals with the dynamics and control of the Chaplygin sphere moving on a plane performing periodic horizontal oscillations. The problems of controlling the motion of spherical bodies with internal propulsion devices (spherical robots) are of much current interest due to the development of robotics. In particular, in [5,6], proofs are given of the complete controllability of the system by means of three rotors, and control torques are presented for cases of motion along various trajectories, including the case where the motion occurs with friction. In [7], the dynamics and control of a spherical robot using two internal rotors is investigated. The dynamics of rigid bodies on a vibrating plane is another developing direction of research. For example, the dynamics of a wobblestone on a vibrating plane with friction is investigated in [8]. Among publications on the dynamics of spherical bodies we mention [9], where the dynamics of the sphere on the surface of complex form performing vibrations is modeled. Ref. [10] investigates the stability and addresses the problems of stabilizing the spin of the sphere with an axisymmetric pendulum rolling on a plane performing vertical vibration.

2. Results and Discussion

This paper considers the rolling of a dynamically asymmetric balanced sphere of mass m and radius ρ on a horizontal plane. It is assumed that the motion of the sphere occurs without slipping and that the supporting plane performs horizontal periodic oscillations with velocity $\dot{\xi}(t)$. The oscillations of the plane are directed along a fixed vector $\boldsymbol{\alpha}$. We also assume that the sphere has gyrostats placed inside it, which do not change the position of the center of mass of the sphere, but generate gyrostatic momentum $\boldsymbol{k}$, which can be used to control the motion of the sphere.

The equations of motion of the Chaplygin sphere on the vibrating plane have the form

$$\tilde{\mathbf{I}}\dot{\boldsymbol{\omega}} + \dot{\boldsymbol{k}} = (\mathbf{I}\boldsymbol{\omega} + \boldsymbol{k}) \times \boldsymbol{\omega} + m\rho\ddot{\xi}(t)\boldsymbol{\beta},$$
$$\dot{\boldsymbol{\alpha}} = \boldsymbol{\alpha} \times \boldsymbol{\omega}, \quad \dot{\boldsymbol{\beta}} = \boldsymbol{\beta} \times \boldsymbol{\omega}, \quad \dot{\boldsymbol{\gamma}} = \boldsymbol{\gamma} \times \boldsymbol{\omega},$$
$$\dot{x}_c = \rho(\boldsymbol{\omega}, \boldsymbol{\beta}) - \dot{\xi}(t), \quad \dot{y}_c = -\rho(\boldsymbol{\omega}, \boldsymbol{\alpha}).$$

where $\tilde{\mathbf{I}}\boldsymbol{\omega} = \mathbf{I}\boldsymbol{\omega} + m\rho^2\boldsymbol{\gamma} \times (\boldsymbol{\omega} \times \boldsymbol{\gamma})$, $\mathbf{I} = \mathrm{diag}(i_1, i_2, i_3)$ is the central tensor of inertia of the sphere, $\boldsymbol{\omega}$ is its angular velocity, x_c, y_c are the coordinates of the point of contact, and $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}$ are the projections of the unit vectors of the fixed coordinate system onto the axes of the moving coordinate system attached to the sphere.

In this paper we show that, in the case of a sphere moving on a plane performing horizontal oscillations, free rolling motion in a straight line is possible only in the direction of oscillations, and the angular velocity of rotation of the sphere is horizontal and perpendicular to the direction of the plane's oscillations.

In this paper we also address the problem of controlling the motion of the sphere on a moving plane. In particular, we consider a number of problems of stabilizing the motion of the sphere by means of gyrostatic momentum $\boldsymbol{k}$ both relative to the oscillating plane and relative to the absolute coordinate system. We have obtained algorithms enabling vertical rotation and uniform motion of the sphere along a straight line relative to the moving and absolute coordinate systems. The study of these problems is motivated by the practical interest in the stabilization of the motion of a spherical robot on a moving plane (for example, inside a moving vehicle).

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