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# Elliptic operators with infinitely many variables

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## 1. Introduction

The extension of the classical theory of linear elliptic PDE's in finite dimensions to infinite dimensional Hilbert spaces H is a widely open field.

In infinite dimensions several well established finite dimensional techniques fail, due for instance to the lack of translation invariant and doubling Borel measures and lack of compactness of bounded closed sets.

These difficulties prevent to study equations by using approximation by convolution with mollifiers and localization methods based on comparison of integrals of functions over balls with integrals over larger balls.

Motivations for studying elliptic equations or parabolic equation in H come from partial stochastic differential equations (SPDEs) arising in different domains as: quantum fields theory, statistical mechanics, biology, chemistry and mathematical finance.

## 2. Results and Discussion

We will be concerned with the following Kolmogorov operator

$$Ku=1//2\ \mathrm{Trace}[CD^2_x u]-\langle Ax-D_xU, D_xu\rangle$$

where C is a positive bounded operator in H, A is a a self---adjoint negative operator and U is a potential.

The SPDE corresponding to K is the following

$$dX=(AX-DU(X))dt+C^{1/2}\,dW(t),\quad X(0)=x$$

where W is a cylindrical Wiener process in a probability space $(\Omega,F,P)$.
We are interested in the evolution equation

$$D_tu=Ku,\ u(0,x)=u_0(x),$$

whose solution is formally given by

$$u(t,x)=E[u_0(X(t,x))],$$

(where the expectation E means integration in $\Omega$, with respect to P) and the corresponding elliptic equation.

In this talk we shall first review some recent results on existence, uniqueness and maximal regularity for elliptic equations in the spaces $L^2(H,\mu)$ where $\mu$ is an invariant measure for the Kolmogorov operator K, see [1]-[4]

Then we shall consider the Cauchy-Dirichlet problem in an open, bounded, convex of H, whose solution is formally given by the probabilistic formula

$$u(t,x)=E[u_0(X(t,x)1_{t<\tau}$$

where $\tau$ is the exit time from the closure of O.

We shall present some new existence and regularity results for the gradient of u.

## References


[1] . G. DA PRATO G, Elliptic operators with infinitely many variables. *Current research in nonlinear analysis*, 2018, 99–127, Springer Optim. Appl., 135, Springer,

[2] G.DA PRATO, A. LUNARDI, Maximal $L^2$ regularity for Dirichlet problems in Hilbert space, J. Math. Pures Appl. 2013, 99, 6, 741--765..

[3] ] G.DA PRATO, A. LUNARDI, Sobolev regularity for a class of second order elliptic PDE's in infinite dimension, *Ann. Probab.* 2014, **42**, 5, 2113--2160.

[4] G.DA PRATO, A. LUNARDI, Maximal Sobolev regularity in Neumann problems for gradient systems in infinite dimensional domain, Ann. Inst. H. Poincar\'e Probab. Stat. , 2015, 42, 5 1102—1123.