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Advanced Complex Plane Field and Differential Equations Approach to Nonlinear Dynamics of the Industrial City's Chaotic Atmospheric Ventilation

OLGA Y KHETSELIUS[1*], ALEXANDER V GLUSHKOV[1], SERGIY M STEPANENKO[1], ANDREY A SVINARENKO[1] AND VASILY V BUYADZHI[1]

1. Odessa State Environmental University, Mathematics Depr., L'vovskaya str. 15, 65009, Odessa
* Presenting Author

Abstract: The paper is devoted to development of an advanced mathematical approach to analysis, modelling and forecasting a chaotic dynamics of industrial city's atmospheric ventilation on the basis of compex geophysical plane field and difernetial equations methods. New advanced version of the Arakawa-Schubert convection model and modified hydrodynamical prediction models are presented. The methods of a plane complex field and spectral expansion algorithms are applied to calculate the air circulation for the cloud layer arrays, penetrating into the territory of the industrial city. The results of the PC simulation experiments for an chaotic air ventilation and a chaotic heat transfer in atmosphere of industrial city, including the data of modelling ventilation (meso-circulation) parameters over Gdansk (Poland) region are presented.

Keywords: nonlinear dynamics, atmospheric ventilation, complex field, differential equations

1. Introduction. Nonlinear Dynamics of Atmospheric Ventilation

Investigation of regular and chaotic energy-, heat-, mass-transfer in continuous mediums and systems is very actual and complex problems of the modern physics of dynamical systems, computational hydrodynamics etc. At present time one could remind about different simplified models that allow to estimate a structure of chaotic air ventilation in an atmosphere. However, these approaches are based on the classical laws of molecular diffusion, as well as the known regression relations models (e.g. [1]) with known disadvantages. More sophisticated approaches such as different versions of the Lagrangian particle dispersion models etc provide significantly more accurate results, however, such approaches require very complicated simulation [2]. Here a novel approach to analysis, modelling and forecasting a chaotic dynamics of industrial city's atmospheric ventilation is presented Sketch for air ventilation between a city and its periphery in a presence of atmosphere convection is in Fig 1.

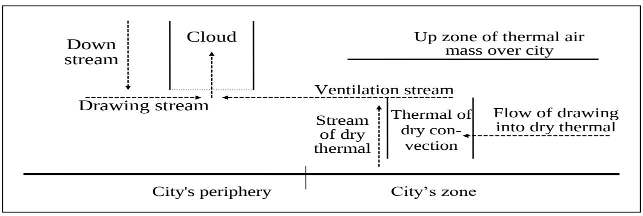

Fig. 1. Sketch for air ventilation between a city and its periphery in a presence of atmosphere clouds convection

2. Model of Nonlinear Dynamics, Results and Discussion

The Arakawa-Schubert model includes the budget equations for mass, moist static energy, total water content plus the equations of motion [1,2]. In the case of air ventilation emergence, mass balance equation in the convective thermalsis as follows [13]:

$$m_B(\lambda) = F(\lambda) + \beta \int_0^{\lambda_{\max}} m_B(\lambda')K(\lambda,\lambda')d\lambda' \tag{1}$$

Here λ is a velocity of involvement, $m_B(\lambda)$ is an air mass flux, $K(\lambda, \lambda')$ is the integral equation kernel, which determines the dynamical interaction between the neighbours clouds; β is parameter which determines disbalance of cloud work due to the return of part of the cloud energy to the organization of a wind field in their vicinity. The solution of Eq. (1) is determined by a resolvent method [1,2]:

$$m_B(\lambda) = F(\lambda) + \beta \int_0^{\lambda_{\max}} F(s)\Gamma(\lambda,s;\beta)ds, \qquad \Gamma(\lambda,s;\beta) = \sum_{i=1}^{\infty} \beta^{i-1} \cdot K_i(\lambda,s) \tag{2}$$

The key idea [2] is to determine the resolvent as an expansion to the Laurent series in a complex plane ζ. Its centre coincides with the centre of the city's "heating" island and the internal cycle with the city's periphery. The external cycle can be moved beyond limits of the urban recreation zone. The Laurent representation for resolventis provided by the standard expansion:

$$\Gamma = \sum_{n=-\infty}^{\infty} c_n(\zeta - a)^n, \qquad c_n = \frac{1}{2\pi i}\oint_{|\zeta|=1} \frac{\Gamma(\zeta)d\zeta}{(\zeta - a)^{n+1}} = \frac{1}{2\pi i}\int_0^{2\pi} \Gamma(e^{it})e^{-int}dt, \tag{3}$$

where a is center of the series convergence. The method for calculating a turbulence spectra inside the urban zone should be based on the standard tensor equations of turbulent tensions (e.g. [1,11]). The velocity components, say, v_x,v_y, of an air flux over the city area are computed in an approximation of "shallow water". The necessary solution for v_x-iv_y component for the city's heat island has a form of expansion into series on the Bessel functions. From the other side, the velocity of an air flux over city's peripheryin a case of convective instability can be found by method of a plane complex field theory in a full analogy with the known Karman vortices chain model [1]. The results of modeling and forecasting [3] the air ventilation parameters for different synoptic situations in the Gdansk (Poland) region are presented. The detailed data about a current function and a velocity potential are listed.

3. Concluding Remarks

To conclude, a new approach to modelling an air ventilation and turbulence in the urban area is developed. The computational data on air ventilation for the typical situations in the Gdansk (Poland) region are presented. New version of the Arakawa-Schubert convection model and modified hydrodynamical prediction models are developed. The methods of a plane complex field and spectral expansion algorithms are applied to calculate the amopsheric circulation prameters.

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