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Border collision bifurcations of chaotic attractors

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Abstract: In 1D piecewise-smooth maps with multiple borders, chaotic attractors may undergo border collision bifurcations, leading to a sudden change in their structure. We describe two types of such border collision bifurcations and explain the mechanisms causing the changes in the geometrical structure of the attractors, in particular in the number of their bands (connected components).

Keywords: piecewise smooth systems, 1D maps, chaotic attractors, border collision bifurcations

1. Introduction

Border collision bifurcations are well-known and represent the main distinguishing feature of piecewise smooth systems [1]. A border collision bifurcation occurs as, under parameter variation an invariant set collides with a border (switching manifold), causing the topological structure of the state space to change. A natural question is, which invariant sets can undergo a border collision bifurcation? For historical reasons, most investigated are border collision bifurcations of fixed points and cycles. As for chaotic attractors, the possibility that they also may undergo a border collision bifurcation has been overlooked for a long time. Indeed, in the most widely investigated class of piecewise smooth 1D maps, namely, in piecewise monotone maps with a single border, a chaotic attractor cannot collide with the border, since if a map belonging to this class has a chaotic attractor, the point of discontinuity is necessarily located inside the attractor [2].

In 1D maps with multiple (at least two) borders, collisions of a chaotic attractor with the borders are possible and may lead to quite interesting bifurcation phenomena, as illustrated in Fig. 1. As one can see, the chaotic attractors are robust in the complete considered parameter range (in the sense of [3], i.e., not interrupted by periodic windows and not affected by coexistence). However, at certain parameter values the geometric structure of the attractor change, additional gaps or bands of the attractors appear. The bifurcation of chaotic attractors known for maps with a single discontinuity (such as merging and expansion bifurcations) are not sufficient to explain the observed structure.

2. Results

In the present work, we investigate bifurcations of chaotic attractors in the piecewise linear map with two discontinuities, defined by

$$x_{n+1} = f(x_n) = \begin{cases} f_{\mathcal{L}}(x_n) &= a_{\mathcal{L}} x_n + \mu_{\mathcal{L}} & \text{if } x < -1, \\ f_{\mathcal{M}}(x_n) &= a_{\mathcal{M}} x_n + \mu_{\mathcal{M}} & \text{if } -1 < x < 1, \\ f_{\mathcal{R}}(x_n) &= a_{\mathcal{R}} x_n + \mu_{\mathcal{R}} & \text{if } x > 1. \end{cases} \quad (1)$$

with $a_L=a_M=a_R=a>1$. Note that the linearity of the branches of map (1) simplifies the calculations but does not restrict the generality of the analysis, so that the obtained results are applicable to any piecewise monotonous everywhere expanding map with two discontinuities.

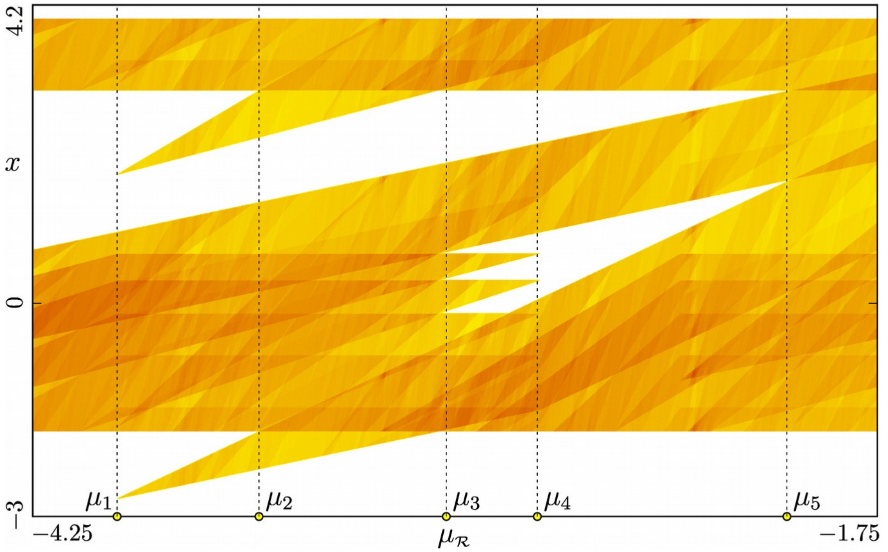

Fig. 1. Bifurcation scenario in map (1) showing exterior (μ_1, μ_4) and interior (μ_2, μ_3, μ_5) border collision bifurcations of chaoric attractors. Parameters: a=1.25, μ_L=4.0, μ_M=4.0,

So far, we have detected two novel types of bifurcations:

- *Exterior border collision bifurcation of a chaotic attractor:* The bifurcation occurs as a chaotic attractor containing one discontinuity point collides with another discontinuity point. As a result of this bifurcation, a number of additional bands of the attractor appear. A distinguishing feature of this bifurcation is that the size of the additional bands shrinks to zero as the varied parameter approaches the bifurcation value.
- *Interior border collision bifurcation of a chaotic attractor:* The bifurcation occurs as a critical point (an image of a discontinuity) located inside a chaotic attractor and possessing exactly two preimages inside the absorbing interval collides with another critical point and one of its preimages disappears. As a result of this bifurcation, a number of additional gaps in the attractor appear. Here, the size of the additional gaps shrinks to zero as the varied parameter approaches the bifurcation value.

It is worth emphasizing that none of these bifurcations of chaotic attractors are associated with homoclinic bifurcations of repelling cycles. This is a striking difference between these bifurcations and other transformations of chaotic attractors, such as merging, expansion and final bifurcations previously reported [2].

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