DSTA 2021
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16th INTERNATIONAL CONFERENCE
Dynamical Systems – Theory and Applications
December 6-9, 2021, On-line

Comparison of selected artificial intelligence algorithms to determine the thermal conductivity coefficient of a porous material

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Abstract: The paper considers the inverse problem of determining the thermal conductivity coefficient of a porous material. The two-dimensional anomalous diffusion equation with Riemann-Liouville fractional derivative is adopted as the model of the direct problem. Presented model (equations and initial-boundary conditions) can be used to describe anomalous diffusion, for example the heat conductivity in porous materials. To solve the direct problem, the finite differences method supplemented by the scheme based on the alternating direction implicit method (ADIM) is used. The input data for the inverse problem are temperature measurements at selected points in the area. Using the input data and the solution of the direct problem, a functional describing the error of the approximate solution is built. Two artificial intelligence algorithms, the ant colony optimization (ACO) and artificial bee colony (ABC), are used to minimize this functional. In the presented numerical example, these algorithms are compared in terms of accuracy, stability and speed of operation.

Keywords: inverse problem, fractional derivative, parameter identification, anomalous diffusion, optimization, artificial intelligence

1. Introduction

The article is divided into three main parts. The first part presents the model and formulates the inverse problem. In this case, the inverse problem consists in selection the thermal conductivity coefficient in such a way that the model output matches the input data for the output problem (values of the state function at selected points in the domain). The second part presents the methods of solving the direct and inverse problems. To solve the direct problem, the differential scheme with alternating direction implicit method was used and the inverse problem is reduced to the search for the minima of the objective function (using swarming algorithms). The last part is devoted to examples and discussion of the results.

In the paper we consider the following model, which is a differential equation with a fractional derivative:

$$c\rho \frac{\partial u(x,y,t)}{\partial t} = \frac{\partial}{\partial x}\left(\lambda_{x1}(x,y)\frac{\partial^\alpha u(x,y,t)}{\partial x^\alpha} - \lambda_{x2}(x,y)\frac{\partial^\alpha u(x,y,t)}{\partial(-x)^\alpha}\right)$$
$$+ \frac{\partial}{\partial y}\left(\lambda_{y1}(x,y)\frac{\partial^\beta u(x,y,t)}{\partial y^\beta} - \lambda_{y2}(x,y)\frac{\partial^\beta u(x,y,t)}{\partial(-y)^\beta}\right) + f(x,y,t), \tag{1}$$

where $(x,y,t) \in \Omega \times [0,T], \lambda_{x1}, \lambda_{x2}, \lambda_{y1}, \lambda_{y2} > 0,$ and $\alpha, \beta \in (0,1)$ are orders of fractional derivatives. To the Equation (1) the initial-boundary conditions are added:

$$u(x,y,t)|_{\partial\Omega} = 0, \quad t \in (0,T]$$

$$u(x,y,t)|_{t=0} = \varphi(x,y), \quad (x,y) \in \Omega$$

Derivatives in the Equation (1) were defined as Riemann-Liouville fractional derivatives:

$$\frac{\partial^\alpha u(x,y,t)}{\partial x^\alpha} = \frac{1}{\Gamma(1-\alpha)}\frac{\partial}{\partial x}\int_0^x (x-s)^{-\alpha} u(s,y,t) ds,$$

$$\frac{\partial^\alpha u(x,y,t)}{\partial(-x)^\alpha} = \frac{-1}{\Gamma(1-\alpha)}\frac{\partial}{\partial x}\int_x^{L_x} (s-x)^{-\alpha} u(s,y,t) ds.$$

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