DSTA 2021 dys-ta.com
16th INTERNATIONAL CONFERENCE
Dynamical Systems – Theory and Applications
December 6-9, 2021, On-line

Dynamics of a multiple-link aerodynamic pendulum

YURY SELYUTSKIY[1*], ANDREI HOLUB[2], CHING-HUEI LIN[3]

1. Lomonosov Moscow State University, Institute of Mechanics, Moscow, Russia [0000-0001-8477-6233]
* Presenting Author
2. Lomonosov Moscow State University, Institute of Mechanics, Moscow, Russia [0000-0001-8982-4097]
3. Chien-Hsin University of Science and Technology, Department of Electrical Engineering, Taoyuan, Taiwan

Abstract: Oscillations of different aeroelastic systems are of interest from the perspective of both practice and theory. One of examples of such systems is an aerodynamic pendulum with several elastically connected links. The influence of different parameters of the pendulum upon the stability of its trivial equilibrium is studied. Oscillations arising in the system are considered for different values of parameters and different number of pendulum links.

Keywords: oscillations, stability, aerodynamic pendulum, aeroelasticity

1. Introduction

Dynamics of multiple link pendulums in gravity field has been investigated be many researchers. Various approaches to formulation of equations of motion and to control of motion of such pendulums were proposed and used, e.g., in [1-3]. In [4], the impact of the last link of a multiple pendulum with a solid surface is studied.

An interesting class of pendulums are aerodynamic pendulums, where the motion the system is determined by aerodynamic forces. These forces are non-conservative and their influence upon the system can be antidissipative or dissipative. Behaviour of single and double aerodynamic pendulums was studied in [5-6]. Here we consider a generalization of this problem to the case of multiple link pendulum.

2. Problem Statement and Discussion

We discuss the behaviour of a multiple aerodynamic pendulum consisting of n links. Each of the first n-1 links represents a weightless rod with a point mass at its end. For simplicity sake, we assume that all these links have the same mass m_1. The last link represents a thin wing with mass m and moment of inertia J (see Fig. 1). All links are connected with similar spiral springs. Axes of all inter-link joints are vertical.

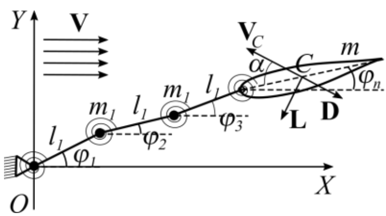

Fig. 1. Scheme of the system

The pendulum is placed in a steady horizontal airflow. It is assumed that the flow interacts only with the wing, and the aerodynamic load is described with the quasi-steady approach [5]. In this context, the aerodynamic load is represented as lift force L and drag D applied in a certain point C of the wind chord, and aerodynamic torque M_z. These values depend on the angle of attack (the angle between the airspeed of point C and the wing chord).

The kinetic energy of the pendulum and the potential energy of springs are as follows:

$$
\begin{aligned}
T &= \frac{1}{2}\sum_{i=1}^{n}\left[\left(m+(n-i)m_1\right)l_i^2\dot{\varphi}_i^2 + 2\sum_{j=i+1}^{n}\left(m+(n-j)m_1\right)l_i l_j \dot{\varphi}_i\dot{\varphi}_j \cos(\varphi_j-\varphi_i)\right] + \frac{J\dot{\varphi}_n^2}{2} \\
U &= \frac{k}{2}\left(\varphi_1^2 + \sum_{i=2}^{n}\left(\varphi_i-\varphi_{i-1}\right)^2\right)
\end{aligned}
\qquad (1)
$$

Here $\varphi_0 = 0$, $l_1 = \ldots = l_{n\text{-}1}$ are lengths of links, l_n is the distance from the last joint to the center of mass of the wing, k is stiffness coefficient of joint springs.

The resulting system of equations of motion is nonlinear and, due to the presence of aerodynamic forces, non-conservative.

Evidently, the system has a trivial equilibrium. Its stability is studied depending on parameters of the system. The effect of the number of links upon the stability and dynamics of the pendulum is considered. Numerical simulation of dynamics of the system is performed for different sets of values of parameters.

3. Concluding Remarks

Detailed analysis of dynamics of a multiple pendulum in airflow could contribute to understanding specific features of behaviour of complicated aeroelastic systems and to development of efficient wind power harvesting devices based on flow-induced oscillations of structures.

References

[1] BRAUN M: On some properties of the multiple pendulum. *Archive of Applied Mechanics* 2003, **72**:899–910.

[2] ANANYEVSKIY I, ANOKHIN N: Control of a multi-link inverted pendulum by a single torque. *IFAC Proceedings Volumes* 2012, **45**(2):550-553.

[3] UDWADIA FE, KOGANTI PB: Dynamics and control of a multi-body planar pendulum. *Nonlinear Dynamics* 2015, **81**:845-866.

[4] MARGHITU DB, ZHAO J: Impact of a Multiple Pendulum with a Non-Linear Contact Force. *Mathematics* 2020, **8**:1202.

[5] LOKSHIN BY, SAMSONOV VA: The self-induced rotational and oscillatory motions of an aerodynamic pendulum. *Journal of Applied Mathematics & Mechanics* 2013, **77**(4):360-368.

[6] SELYUTSKIY YD, HOLUB AP, DOSAEV MZ: Elastically mounted double aerodynamic pendulum. *International Journal of Structural Stability and Dynamics* 2019, **19**(5):1941007