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Estimation the Domain of Attraction for a System of Two Coupled Oscillators with Weak Damping,

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Abstract: When solving a wide class of problems of nonlinear dynamics, the stability property of a given system regime is a prerequisite for design. An important role is played by the concept of the domain of attraction (DoA) of the equilibrium point (or limit cycle). However, as a rule, this domain is difficult to find and describe in explicit form. Therefore, the search for DoA estimate has been a fundamental problem in the control theory since the middle of the last century. Currently, methods based on Lyapunov functions predominate in the literature. We have studied the problem of obtaining the estimates of the DoA for equilibrium of the mechanical systems. The method for using Lyapunov function of special kind for a system with polynomial right-hand side to find the estimates for DoA is proposed. This procedure is illustrated by the example of a mechanical system which consists of two coupled nonlinear oscillators with weak damping.

Keywords: domain of attraction, nonlinear oscillator, weak damping.

1. Introduction

One of the fundamental problems in engineering is proving that performance limits and operational restrictions are met. From the standpoint of the theory of dynamical systems, this problem can be considered as a problem of ensuring the local asymptotic stability on a suitable subset of the state space called the attraction domain or region of attraction (RoA). This approach is used in a variety of control applications such as aerospace [1], power systems [2], chemistry [3], medicine [4] and others. Existing approaches for approximating the DoA can be divided into Lyapunov and non-Lyapunov based categories. Among the non-Lyapunov based approaches (which do not employ explicitly Lyapunov functions) it should be noted papers [5-7]. Today it is a great variety of Lyapunov based methods for estimating the DoA (see, for instance, [8, 9] and references in [10]). Mostly, they are based on the search for a Lyapunov function *V(x)* and for a positive scalar *c*, such that $\dot{V}(x)$ is negative over the sub-level $C = \{x : V(x) \le c\}$. Given such V and c, it can be shown that the connected component of C containing the equilibrium is an inner approximation to the DoA.

In the present paper, we propose a method for using Lyapunov function of special kind for a system with polynomial right-hand side to find the estimates for DoA. This approach is applied to obtain

the estimates of the DoA for equilibrium of a mechanical system which consists of two coupled nonlinear oscillators with weak damping.

2. Results and Discussion

A mechanical system which consists of two oscillators with nonlinear coupling is considered. The dynamics of such a system is described by the following equations

$$\begin{aligned} m_1 \ddot{x}_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_1 x_1 + k_2^{lin}(x_1 - x_2) - k_2^{nonlin}(x_1 - x_2)^3 = 0, \\ m_2 \ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2^{lin}(x_2 - x_1) - k_2^{nonlin}(x_2 - x_1)^3 = 0. \end{aligned} \tag{1}$$

If $k_2^{lin} > 0$, then equilibrium $x = 0, \dot{x} = 0$ is asymptotically (exponentially) stable. The aim of the paper is to obtain the effective estimation for DoA in assumption that mechanical parameters of the system are known. The accompanying task is to study the influence of variation in linear stiffness ratio and damping coefficient onto size of the resulting estimate. To fulfil the task, the Lyapunov function based approach described in [11] is employed. These estimations are compared with the results of numerical integration of the system (1).

3. Concluding Remarks

In this paper, we deal with the problem of estimating the domain of attraction (DoA) for equilibrium of a mechanical system which consists of two coupled nonlinear oscillators with weak damping. For this purpose, we have successfully established a procedure to determine the Lyapunov function of special kind for a system with polynomial right-hand side.

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