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Analysis of the influence of tyre cross-sectional parameters on the stability of a nonlinear bicycle model with elliptic toroidal wheels

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Abstract: In this work, the stability of a bicycle with elliptic toroidal wheels is analysed in detail. The influence of the tyre cross-sectional parameters on the stability range of the rectilinear motion with constant speed is studied. The bicycle multibody model is based on a well-acknowledged bicycle benchmark, which has been extensively used both for theoretical and experimental works. The nonlinear equations of motion, which correspond to a differential-algebraic system of equations, are derived and linearized along the forward upright motion, allowing the expression of the resulting Jacobian matrix as function of the tyre cross-sectional parameters. With this, a sensitivity analysis of the eigenvalues with respect to the geometric parameters of the wheels is performed. The velocity range for which the bicycle is stable in the rectilinear motion with constant velocity is obtained for different scenarios, and the influence of the elliptic section of the toroidal wheel is illustrated with various stability regions.

Keywords: stability, multibody, sensitivity analysis, bicycle benchmark, toroidal wheel

1. Introduction

The stability of bicycles has been widely studied over the years. Meijaard et al. [1] proposed a detailed benchmark bicycle model, whose linear stability along straight and circular motions with constant velocity was thoroughly analysed in Refs. [2]-[4]. In this work, the wheels of the bicycle benchmark are modelled as two elliptic tori instead of hoop-shaped wheels. The results show that the tyre cross-sectional parameters have significant influence on the stability of the forward upright motion.

2. Methodology

The equations of motion of the bicycle multibody model, with holonomic and nonholonomic constraints, constitute a nonlinear index-3 differential-algebraic system of equations [4]. The wheels of the bicycle, which are assumed to roll without slipping, are modelled as two tori of major and minor radii ρ_i and a_i, see Fig. 1 (b). The cross-sections of the wheels are elliptic (see Fig. 1), being a_i and b_i the semi-major and minor axes of the elliptic wheel cross-section, respectively. The elliptic profile r_i, in polar form relative to its centre, is given by:

$$r_i(\eta_i) = \frac{a_i b_i}{\sqrt{\left(b_i \cos(\eta_i)\right)^2 + \left(a_i \sin(\eta_i)\right)^2}}, \tag{1}$$

where η_i is the angular coordinate, measured from the major axis. The geometry of the tyre is defined by the nondimensional parameters $\mu = \frac{a_i}{\rho_i}$ and $\sigma = \frac{b_i}{a_i}$.

3. Results and Discussion

The self-stability range of the uncontrolled bicycle benchmark model is $v_w < v < v_c$, where v_w, v_c are the weave and capsize speeds, respectively [1]. The model of the present work is validated with the results of [1] by particularizing the elliptic toroidal tyres for $\sigma = 0$ (limit case corresponding to the hoop-shaped wheels), which leads to the evolution of the eigenvalues with the forward speed shown in Fig. 2 (a). The stability regions $\sigma - v$ of Fig. 2 (b), corresponding to $\mu = 0.05$, $\mu = 0.1$ and $\sigma \in [0.6, 1]$, show that the inclusion of torus-shaped wheels with elliptic section greatly impacts on this self-stability range, being highly sensitive to the cross-sectional parameters of the tyre.

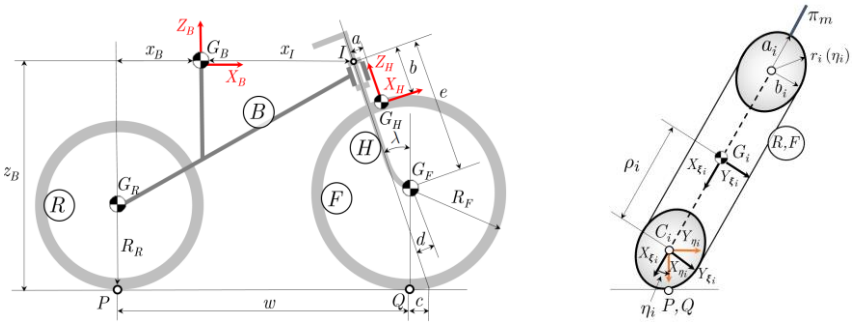

Fig. 1. Lateral view of the bicycle model (a) and front view of the toroidal wheels with elliptic cross-section (b).

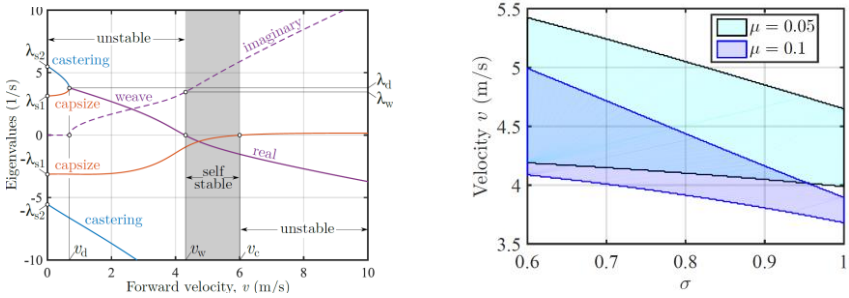

Fig. 2. Evolution of the eigenvalues with v (a) and stability regions $\sigma - v$ for $\mu = 0.05$, $\mu = 0.1$ (b)

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