2021 DSTA dys-ta.com
16th INTERNATIONAL CONFERENCE
Dynamical Systems – Theory and Applications
December 6-9, 2021, On-line

Analytical and finite element models of nonlinear dynamic behaviour of bi-material beam

SIMONA DONEVA[1,2*], JERZY WARMINSKI[1], EMIL MANOACH[2]
[1]*Department of Applied Mechanics, Lublin University of Technology, Lublin, Poland*
[0000-0002-5386-9063], [0000-0002-9062-1497]
[2]*Institute of Mechanics, Bulgarian Academy of Sciences, Sofia, Bulgaria*
[0000-0002-9571-8618]

Keywords: bi-material beam, nonlinear vibrations, Timoshenko beam theory, thermoelasticity, finite element modelling.

Abstract:

The goal of the present work is to develop a theoretical model allowing analytical and numerical study of bi-materials beams subjected to dynamic mechanical and thermal loads. The model is based on geometrically nonlinear version of the Timoshenko beam theory. The extended equations of motions of vibrating beams at elevated temperature are studied analytically by the harmonic balance method and resonance curves for different parameters of the mechanical and thermal loading are obtained. The coupled vibrations of the beam are studied by using 3D finite element analysis.

1. Introduction

Being used in so many technological areas, the bi-material beams are often subjected to mechanical and thermal loading. In many cases such loads lead to large, geometrically nonlinear vibrations. The problems of thermoelastic vibrations of structures are studied by many authors. Nonlinear dynamics of composite plate have been studied in [1], suggesting a temperature distribution along the plate thickness. In [2] thermo-mechanical, geometrically nonlinear vibrations of plates are studied. The authors found very reach nonlinear dynamic behaviour of the system Uncoupled and coupled vibrations of bi-material beams were recently studied by numerical approaches in [3].
In the present work the dynamic behaviour of a bi-material beam subjected to harmonic forces and thermal loading (elevated temperature or heat flux acting on the beam surface) is analysed by using numerical and analytical approaches.

2. Problem formulation and research methods

Nonlinear thermoelastic vibrations of a clamped-clamped Timoshenko beam are studied in the paper. The mathematical model of the beam is derived taking into account geometric nonlinearities. Using the classical dynamic equilibrium method, the differential equations of motion of the bi-material Timoshenko beam are defined. The problem is studied using two different approaches. In the first approach a three-dimensional finite element model of the beam is created using the commercial software ANSYS. The nonlinear vibrations of a heated beam, as well as the vibration of the beams subjected to heat flux are analysed. It is shown that the increased temperature has a significant impact on the beam's reaction and it can cause the beam to oscillate in a complex way (see **Fig.1**).

The second approach is based on a reduced model of the beam dynamics. The Galerkin approach is used to transform the nonlinear partial differential equations into ordinary differential equations. Then the harmonic balance method (HBM) is applied to the reduced model, taking into account the first vibration mode. By this method the reduced nonlinear one-degree of freedom model with cubic non-linearity and temperature influence is studied analytically. By the HBM the resonance curves are determined from the analytically obtained modulation equations and the stability analysis is done.

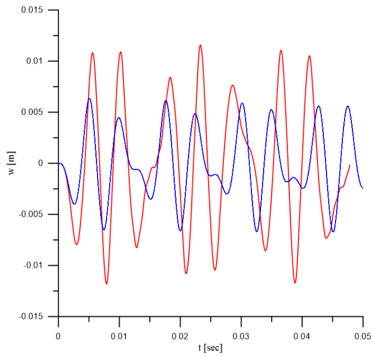

Fig. 1. Time history of the beam centre subjected to uniformly distributed load harmonic load $p=p_0\sin(\omega_e t)$ with p_0=105 kPa and ω_e= 160.78 Hz. obtained by 3D FE model . Blue line – ΔT=0, red line – ΔT=50

3. Conclusions

The paper investigates the bi-material beam model resulting from the extended nonlinear Timoshenko beam theory. The dynamic behavior of a geometrically nonlinear Timoshenko beam model under the combined action of mechanical and thermal load, studied by two different approaches shows a good agreement. It has been shown that the increase of thermal loading also for low mechaical excitations may lead to system's complex dynamics.

Acknowledgments:

The first author acknowledges the financial support by a project "International third-cycle studies in the discipline of Mechanics carried out at the Lublin University of Technology" no. POWR.03.02.00-00-I017 / 16

The first and the third author are grateful for the financial support from the Bulgarian Ministry of Education and Science, Grant No. D01-221/03.12.2018 for NCDSC – part of the Bulgarian National Roadmap on RIs.

References

[1] SAETTA E, REGA G, Unified 2D continuous and reduced order modeling of thermomechanically coupled laminated plate for nonlinear vibrations. *Meccanica 49 (8)*, 1723–1749 ,2014

[2] AMABILI M, CARRA S, Thermal effects on geometrically nonlinear vibrations of rectangular plates with fixed edges, *Journal of Sound and Vibration* 321, 2, 936-954 ,2009

[3] MANOACH E, DONEVA S, WARMINSKI J, Coupled, thermo-elastic, large amplitude vibration of bi-material beams, in H. Altenbach et al. (eds) , *Analysis of Shells, Plates and Beams, Advanced Structured Materials* 134, pp 227-242. Springer Nature Switzerland AG 2020