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The Effect of Initial Stress on Nonlinear Lateral Vibrations of Rotating Rods

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Abstract: In this work we study nonlinear lateral vibrations of a simply supported rotating rod under the effect of initial stress and external loadings. The foundations of Novozhilov's nonlinear theory of elasticity and Biot's general theory for initially stressed solids are used when deriving a generalized elastic potential, based on which the nonlinear mathematical model is developed. The Bubnov-Galerkin approach and Wolfram Language built-in numerical method are utilized to find a solution of the model. The effect of the initial stress field on the rod spatial vibrations is analyzed and graphs for various values of the initial stress are plotted.

Keywords: nonlinear mathematical model, initial stress, rod, lateral vibration

1. Introduction

It is known that the stress state of various parts of machines, building structures and many physical and mechanical systems can be characterized by existence of a field of initial stresses even if there are no external forces affecting them. Fundamental equations of media with initial stresses were obtained by Trefftz, Neuber, Green and Zerna on the basis of the general theory of elasticity for finite deformations [1], and the original theory of nonlinear elasticity for initially stressed solids and fluids was created by Biot [2]. As stated in [3], when examining wave propagation in bodies with initial stresses, the nonlinear theory has to be utilized. At the same time, the initial stress has a dominant role only in the bending modes [4]. Therefore, this work focuses on studying the effect of the field of initial stresses on the spatial deformation of an elastic medium taking into account geometric nonlinearity and external loads.

2. Mathematical Model

Consider a pre-stressed deformable isotropic elastic rod of length l with constant cross-section, rotating around the rod axis z with angular speed ω and being under the action of a longitudinal compressive load $N(z,t)$ and a torque $M(z,t)$. It is assumed that the rod vibrations $u(z,t)$ and $v(z,t)$ take place in the *Oxz*- and *Oyz*-planes, respectively. Based on the concepts of Novozhilov's nonlinear elasticity theory [5] and Biot's general theory, a generalized elastic potential accounting for initial stress is derived. Applying the Ostrogradsky-Hamilton variation principle, a nonlinear mathematical model of spatial lateral vibrations of the simply supported rotating rod under the effect of the initial stress field σ_{ij}^{0} and external loads is developed:

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$$
\begin{aligned}
&\rho A\frac{\partial^2 u}{\partial t^2}+EI_y\frac{\partial^4 u}{\partial z^4}-\rho I_y\frac{\partial^4 u}{\partial z^2\partial t^2}+\frac{\partial^2}{\partial z^2}\left(M(z,t)\frac{\partial v}{\partial z}\right)+\frac{\partial}{\partial z}\left(N(z,t)\frac{\partial u}{\partial z}\right)-\frac{EA}{1-\nu}\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\right)^3\\
&\quad-\frac{EA(5-6\nu)}{2(1-\nu)}\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial z}\left(\frac{\partial v}{\partial z}\right)^2\right)-\rho A\left(2\omega\frac{\partial v}{\partial t}+\omega^2 u\right)+\frac{A}{2}\left(\sigma_{yy}^0+\sigma_{zz}^0\right)\frac{\partial^2 v}{\partial z^2}+A\sigma_{xy}^0\frac{\partial^2 u}{\partial z^2}=0,\\
&\rho A\frac{\partial^2 v}{\partial t^2}+EI_x\frac{\partial^4 v}{\partial z^4}-\rho I_x\frac{\partial^4 v}{\partial z^2\partial t^2}-\frac{\partial^2}{\partial z^2}\left(M(z,t)\frac{\partial u}{\partial z}\right)+\frac{\partial}{\partial z}\left(N(z,t)\frac{\partial v}{\partial z}\right)-\frac{EA}{1-\nu}\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial z}\right)^3\\
&\quad-\frac{EA(5-6\nu)}{2(1-\nu)}\frac{\partial}{\partial z}\left(\frac{\partial v}{\partial z}\left(\frac{\partial u}{\partial z}\right)^2\right)+\rho A\left(2\omega\frac{\partial u}{\partial t}-\omega^2 v\right)+\frac{A}{2}\left(\sigma_{xx}^0+\sigma_{zz}^0\right)\frac{\partial^2 u}{\partial z^2}+A\sigma_{xy}^0\frac{\partial^2 v}{\partial z^2}=0.
\end{aligned}
\qquad (1)
$$

3. Numerical Results

For finding a solution to the nonlinear model (1), the Bubnov-Galerkin approach, according to which the lateral displacements $u(z,t)$ and $v(z,t)$ are approximated by a finite spectrum of harmonic modes, and Wolfram Language built-in numerical method giving results in terms of interpolating functions are applied. To get a general picture of the oscillatory process and estimate the influence of initial stress, the graphs of the rod spatial lateral vibrations are constructed. Considering cases when the initial stress components are equal (Fig. 1) and have different values, along with examining compressive and tensile stresses allow studying the effect of the initial stress field on the rod lateral vibrations taking into account its rotation and the external loadings.

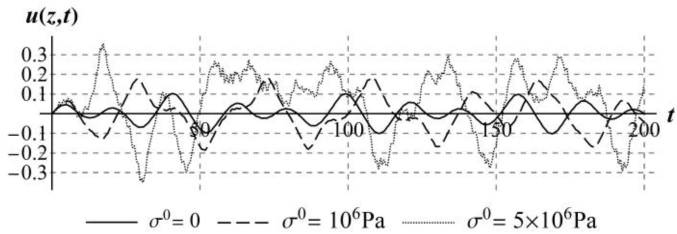

Fig. 1. The effect of initial stress on the rod lateral vibrations in the *Oxz*-plane

4. Concluding Remarks

In this paper we estimated the effect of initial stress on nonlinear lateral vibrations of a rotating rod under the action of an external compressive load and a torque. Accounting for the tensile initial stresses allowed reducing the amplitude of the rod vibrations, whereas the compressive stresses resulted in their increase with retaining stability of the oscillatory process at optimal system parameters.

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