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Dynamics of Rotating Cylindrical Shell Subjected to Pressure Loading

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Abstract: This study pertains to the dynamics of a rotating cylindrical shell and the effect of follower pressure force acting. We consider Donnell's thin shell theory. The considered structure is gyroscopic and is acted upon by follower forces in the form of pressure. The presence of this non-conservative force in this gyroscopic system exhibits interesting dynamical behavior and this abstract provides an overview.

Keywords: Donnell's shell theory, critical speed, gyroscopic system, follower forces

1. Introduction

Thin cylindrical structures are one of the most significant and widely utilised components in various industrial applications such as the aviation industry, electrical system, pressure vessels, nuclear industry and offshore excavation sites. The wide range of applicability of thin cylindrical structures has compelled several authors to study their dynamics in the past. In several applications associated with the offshore oil mining industry, biological industry [1], the thin cylindrical structures undergo pressure loading, and in some applications (offshore drilling), they experience continuous rotation. Hence, the study of the dynamic behaviour of the rotating shell is relevant.

In the past, several researchers have investigated the dynamics of a thin rotating cylinder. Recent studies by Carrera et. al. [2] consider vibration analysis of cylindrical shells using a refined beam model resulting in accurate results for lower frequencies. The accuracy for higher frequencies can be further increased by using a three-dimensional cylindrical shell model. In the past, a numerical study of three-dimensional cylindrical shell based on hierarchical finite element model of a non-rotating thin cylinder has been carried out by Paulo et. al. [3], where higher-order modes have been analysed using a set of reduced-order equations. By considering Donnell's thin shell model, Alujevic et. al. [4] and Ng et. al. [5] have studied the effect of cylinder rotation on the normal modes.

In several applications, pressurised shells are used, where the shell displacements become finite in amplitude resulting in displacement dependent pressure load [6]. We herein consider the free and forced dynamics of a thin rotating (angular velocity Ω about x axis) cylindrical shell of length L, radius R, thickness t and with a constant internal pressure p. The constant pressure load is modelled as a displacement dependent pressure load [6]. The nondimensional equations of motion invoking the Donnell Mushtari's shell theory is given by

$$u_{xx} + \left(\frac{1-\mu}{2}\right) u_{\theta\theta} + \left(\frac{1+\mu}{2}\right) v_{x\theta} + \mu w_x + \Omega^2 (u_{\theta\theta} - w_x) - \frac{p}{\beta} w_x = u_{tt} \tag{1a}$$

$$\left(\frac{1}{2} + \frac{\beta^2}{24}\right)(1-\mu) v_{xx} + \left(1 + \frac{\beta^2}{12}\right) v_{\theta\theta} - \frac{\beta^2}{12} w_{3\theta} - \frac{\beta^2}{12} w_{xx\theta} + \left(\frac{1+\mu}{2}\right) u_{x\theta} + w_\theta + v\Omega^2 + 2\Omega w_t + \Omega^2 u_{x\theta} - \frac{p}{\beta} w_\theta = v_{tt} \tag{1b}$$

$$-\frac{\beta^2}{12} w_{4x} - \frac{\beta^2}{12} w_{4\theta} + \frac{\beta^2}{12} v_{\theta\theta\theta} - \frac{\beta^2}{6} w_{xx\theta\theta} + \frac{\beta^2}{12} v_{xx\theta} - \mu u_x - v_\theta - w + w\Omega^2 - 2\Omega v_t + \Omega^2(-v_\theta + w_{\theta\theta}) + \frac{p}{\beta}(1 + u_x + v_\theta + w) = w_{tt} \tag{1c}$$

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Where u, v, w are the shell deformations in x, θ and r directions respectively, μ is Poisson's ratio, $\beta = h/R$ and $f_{\theta\theta\theta\theta} = \partial^4 f/\partial\theta^4$. The boundary conditions considered in here are simply supported.

2. Results and Discussions

In this study, the variation of natural frequency $\omega^{(m,n)}$ of the cylindrical shell with angular velocity and internal pressure is studied, where m is the number of half waves in axial direction and n is the number of circumferential waves. For the first circumferential mode i.e., $n = 1$, the natural frequency becomes zero for specific angular velocities, which are the critical speeds. In contrast, $n \neq 1$ does not exhibit such critical speeds. For $m = 1, n = 1$ an increase in pressure increases the critical speed as shown in Fig. 2a. In contrast, for $m = 1, n = 2$, no such critical speeds are observed in Fig. 2b. As observable, the follower pressure force has a significant effect of the natural frequencies.

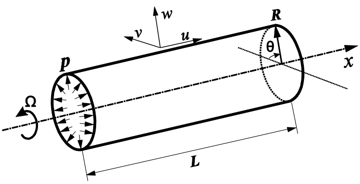

Fig.1 Kinematic description of the cylindrical shell

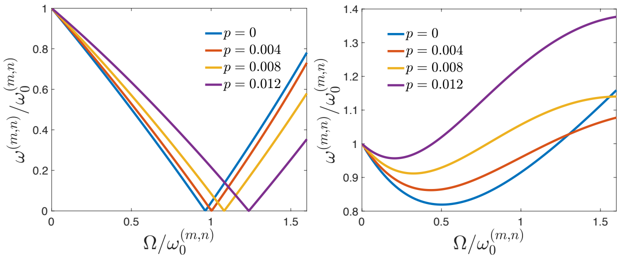

Fig. 2. Normalized angular velocity v/s normalized natural frequency for varying pressure for (a) $m = 1, n = 1$ (b) $m = 1, n = 2$, $\omega_0^{(m,n)}$ is the natural frequency for $\Omega = 0$. $\mu = 0.3$, $\beta = 0.02$, $\alpha = 6$.

3. Concluding Remarks

In this work, we study the effects of internal pressure and angular velocity on the normal modes of the thin rotating cylindrical shell. Exact analytic solutions for the vibrations of a cylindrical shell with simply supported boundary conditions are obtained. It is found that for the modes with $n = 1$, a critical speed is obtained, which results in a zero natural frequency. It is observed that the critical speed increases with increase in internal pressure. A complete analysis will be reported in the full version of the paper.

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