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Forced vibrations in a dynamic system equipped with a mechanism which trans-pass through its singular position

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Abstract: The paper is about modelling of dynamics and about vibrations of hybrid systems composed of a continuous elastic beam and of a multibody system. To merge the two parts, constraint equations are used. Since number of constraint involved coordinates is small, a case dedicated algorithm is proposed to eliminate Lagrange's multipliers and dependent coordinates. The applicational focus is set to amplify damping. To amplify it for the lower-frequency modes, inter-modal energy transfer is enforced due to nonlinearity of dynamics of the multibody part. Proposed amplification becomes effective if the multibody part is put in a shape which is close to its kinematical singular position. Focusing on free vibrations, efficiency of the damping method was verified successfully in few of the previous works of the Author. The present work investigates effectiveness of damping of forced vibrations.

Keywords: Finite elements, multibody, nonlinear vibrations, damping amplification, singularity

1. Introduction

Attention of the paper is set on dynamics and on vibrations of a hybrid system composed of a continuous elastic element and of a multibody system (Fig. 1a). Since dynamics properties of the last are position dependent, resulting system may not be treated as constant linear vibrating system. From the computational point of view, two aspects are crucial. Firstly, right coupling have to be formulated between the linear finite elements model of the beam and the nonlinear model of the multibody part. Secondly, Lagrange's multipliers have to be added, and then, the lasts and the dependent coordinates have to be eliminated. Since only few of the system coordinates are involved in the constraints, a case dedicated algorithm is recommended to reduce the numerical cost of the elimination.

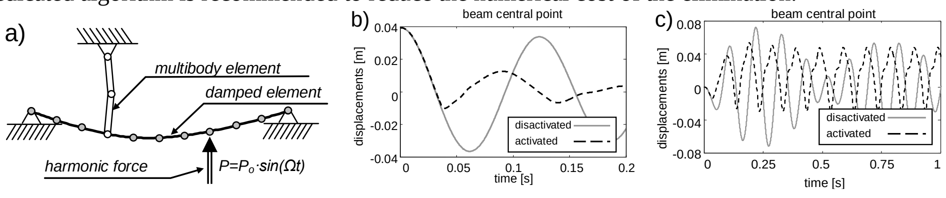

Fig. 1. Draft of the investigated system (a); motion of a selected node – free vibrations (b); motion of the same node – forced vibrations (c)

From the applicational point of view, the focus is set on damping amplification. Structural damping is implemented in the continuous element, thus higher frequency modes are well damped. To accelerate damping of lower-frequencies modes, inter-modal energy transfer is enforced due to the presence of the multibody part. Modal disparity is observed, when motions are performed at different deformations of the beam. Amplification becomes especially crucial (Fig. 1b) when the multibody part is set in its pose which is close to its kinematical singular position [1-3]. But numerical problems are associated with such configuration, e.g., if joint kinematics is searched for a given vertical kinematics of the fixing point, determinant of the Jacobian matrix tends to zero at the singularity. Thus, if small random error of estimation of the vertical position is assumed, it effects in a significant random error of joint positions and in lock of the numerical integration procedures in consequence.

2. Modelling

To test the damping, related numerical model is introduced. Modelled system is planar (fig. 1a), and it consists of two parts: of an elastic beam and of a multibody part. The multibody part is composed of two rigid bodies and of two massless joints. Joint displacements are considered as its generalized coordinates. Free body diagrams are plotted and associated dynamics equations are written. Final matrix form of dynamics is written with use of the generalized coordinates, $\boldsymbol{q}^{\boldsymbol{b}}$. It leads to [1-3],

$$\boldsymbol{M}^b(\boldsymbol{q}^b)\cdot\ddot{\boldsymbol{q}}^b+\boldsymbol{F}^b(\dot{\boldsymbol{q}}^b,\boldsymbol{q}^b)-\boldsymbol{Q}^b(\dot{\boldsymbol{q}}^b,\boldsymbol{q}^b,\boldsymbol{f}^e,\boldsymbol{t}^e,t)=0 \tag{1}$$

To express dynamics of the elastic beam, finite elements are used. Deformations of nodes are considered as the generalized coordinates of the beam, $\boldsymbol{q}^{\boldsymbol{c}}$. Resulting dynamics equation is [4],

$$\boldsymbol{M}^c\cdot\ddot{\boldsymbol{q}}^c+\boldsymbol{D}^c\cdot\dot{\boldsymbol{q}}^c+\boldsymbol{K}^c\cdot\boldsymbol{q}^c=\boldsymbol{P}^c\ . \tag{2}$$

To merge the systems, constraint equations are used. The end-point of the last arm is joined to the beam by use of spherical joint constraint. Beam connected point coincides with a node of its FEM model. Constraints are written at the position level, and then formulae of their time derivatives are used. Next, dynamics equations Eqs. (1) and (2) are supplemented with Lagrange's multipliers [1-3]:

$$\boldsymbol{M}^b\cdot\ddot{\boldsymbol{q}}^b+\boldsymbol{F}^b-\boldsymbol{Q}^b+\boldsymbol{J}^{b^T}\cdot\boldsymbol{\lambda}=0\,;\qquad \boldsymbol{M}^c\cdot\ddot{\boldsymbol{q}}^c+\boldsymbol{D}^c\cdot\dot{\boldsymbol{q}}^c+\boldsymbol{K}^c\cdot\boldsymbol{q}^c+\boldsymbol{J}^{c^T}\cdot\boldsymbol{\lambda}=\boldsymbol{P}^c\ . \tag{3}$$

Then, Lagrange's multipliers and the dependent coordinates are eliminated. A modified version of the classical coordinate partitioning [3] is used in order to eliminate some of the non necessary zero numerical calculations. To reduce numerical problems resulting of singularity, a coordinate of the multibody part is set as the independent parameter. After the elimination the dynamic equations are,

$$\boldsymbol{M}^*\cdot\ddot{\boldsymbol{q}}^*+\boldsymbol{F}^*-\boldsymbol{Q}^*=0\ . \tag{4}$$

3. Results, Discussion and Concluding Remarks

Focusing on free vibrations of the elastic/multibody system, efficiency of the damping method was verified successfully [1,2] (Fig. 1.b). The presently taken tests are focused on forced vibrations. The tests have verified that the method is less effective in the forced case (Fig. 1.c). Reduction of amplitudes is confirmed, however, its range is low. Additional tests and analyses are recommended.

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