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A dynamic-stiffness framework for locally resonant structures

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Abstract: A comprehensive analytical/numerical framework is presented for locally resonant metamaterial structures, modelled as continuous beams coupled with periodic multi-degree-of-freedom subsystems representing the resonators. An exact reduced-order dynamic-stiffness approach is formulated, where the beam equations are solved exactly upon condensing all the degrees of freedom within the resonators. Solutions to the free-vibration problem are obtained from an exact dynamic-stiffness matrix, whose size is 4×4 for any number of resonators. Solutions to the forced vibration problem under arbitrary loads are obtained by modal superposition, in time and frequency domains, on introducing proper orthogonality conditions for the modes. The framework is suitable for Euler-Bernoulli or Timoshenko beams and typical resonators as mass-spring-dashpot chains, mass-spring-dashpot systems within rigid truss assemblies, inerter-based absorbers. Viscous damping within the resonators can be considered.

Keywords: metamaterial, locally resonant structure, dynamic-stiffness method

1. Introduction

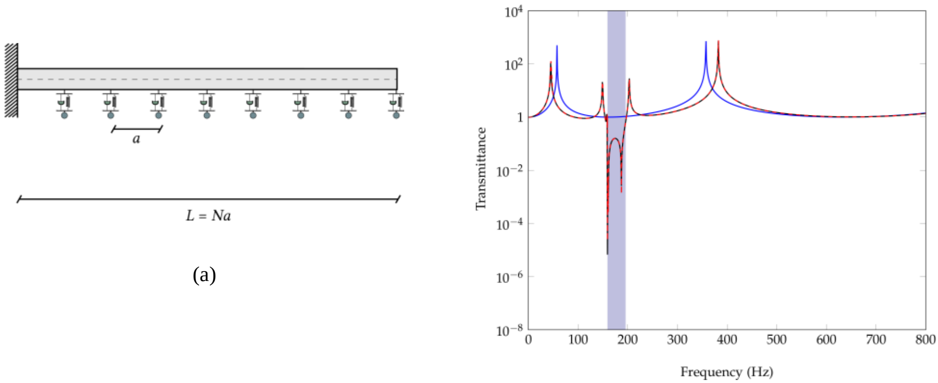
Locally resonant metamaterial structures are an emerging concept in dynamics, with promising applications in many real-life engineering applications. The term “locally resonant structure” refers to a beam or plate coupled with a periodic array of multi-degree-of-freedom resonators where, as a result of the periodic resonance induced by the resonators, elastic waves cannot propagate over stop bands named *band gaps*. For this remarkable property, locally resonant structures attracted a great deal of attention in the last decade, as demonstrated by the outstanding number of works on this topic [1,2].

Here, we focus on locally resonant structures consisting of continuous beams coupled with multi-degree-of-freedom resonators. We promote an exact reduced-order dynamic-stiffness formulation, where the beam equations are solved exactly upon condensing all the resonator degrees of freedom. The formulation delivers an exact dynamic-stiffness matrix of size 4×4 for any number of resonators, from which the solution to the free-vibration problem is obtained. Further, the modal response under arbitrary loads is obtained in a concise analytical form, in time and frequency domains, deriving proper orthogonality conditions for the modes of the beam only. The proposed framework is general, as applies to various beam models (Euler-Bernoulli and Timoshenko) and various resonators, as mass-spring-dashpot chains, mass-spring-dashpot-truss assemblies, inerter-based absorbers.

2. Results and Discussion

We consider the locally resonant Timoshenko beam in Fig. 1a, coupled with 1-degree-of-freedom mass-spring-dashpot resonators. Parameters of beam are E = 88.5 MPa, A = 0.08 m^2, ν = 0.30, ρ = 2700 kg/m^3, κ = 0.85 (shear correction factor); $k_r = 2 \times 10^5$ N/m, m_r = 0.2 kg, c_r = 0.03 Ns/m, a = 0.1

m and $N = 8$ are taken for the resonators. We calculate complex modes in Table 1 and transmittance of the beam in Fig. 1b by the proposed dynamic-stiffness matrix (DSM) approach (black continuous line), the finite-element method (FEM) in Abaqus (red dashed line); the exact response without resonators (blue continuous line) is included.



(a)

(b)

Fig. 1. Locally resonant Timoshenko beam with 1-degree-of-freedom mass-spring-dashpot resonators

Table 1. Complex eigenvalues of the beam in Fig. 1

Mode	Eigenvalue (DSM)	Eigenvalue (FEM)
1	± 283.39 + 0.001	± 283.39 + 0.001
2	± 937.19 + 0.056	± 937.19 + 0.056
3	± 992.53 + 0.073	± 992.54 + 0.073
4	± 998.01 + 0.074	± 998.01 + 0.074
5	± 999.22 + 0.075	± 999.23 + 0.075
6	± 999.61 + 0.075	± 999.61 + 0.075
7	± 999.76 + 0.075	± 999.76 + 0.075
8	± 999.82 + 0.075	± 999.82 + 0.075
9	± 1274.64 + 0.117	± 1274.65 + 0.117
10	± 2401.47+ 0.064	± 2401.71 + 0.064

3. Concluding Remarks

We will formulate an exact reduced-order dynamic-stiffness framework for locally resonant metamaterial beams, addressing the free and forced vibration problems.

A relevant aspect of the framework is that it handles various beam models and resonators. Furthermore, we will demonstrate the framework is not applicable to beams only, as generalizations are readily feasible for locally resonant metamaterial Kirchhoff plates.

References

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