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Cloaking of Love Waves

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Abstract:

We present the design of an elastic carpet cloak capable of rerouting Love waves around a defect. Love waves are horizontally polarized surface waves that propagate in a soft elastic medium overlaying a stiffer substrate. Their governing equation has the form of the scalar Helmholtz equation. Building upon the invariance of the Helmholtz equation under an arbitrary coordinate transformation, we utilize the principle of transformation elastodynamics to design a surface cloak for Love waves. We evaluate via numerical simulations the validity of our approach by cloaking a triangular-shaped defect that lies within the soft layer. We show that the designed cloak is capable to hide the surface defect by generating near zero scattered field.

Keywords: Love Waves, Cloaking, Transformation Elastodynamics, Metamaterials.

1. Introduction

The prospect of rerouting the propagation of elastic waves around an object and isolate it from unwanted mechanical vibrations have fuelled the research interest towards the realization of elastic cloaking devices. In the past decades, several works proposed theoretical models and experimental realization of cloaking devices for bulk waves, flexural waves in plates or antiplane waves. Cloaking theory can be traced back to [1]. Surprisingly, applications of cloaking for elastic surface waves are scarce. In this work, we explore the possibility of cloaking surface waves of the Love type.

2. Governing Equations

We consider a homogeneous, isotropic elastic layer of thickness h_1 and material properties (λ_1, μ_1, ρ_1) coupled to a semi-infinite medium with properties (λ_2, μ_2, ρ_2) where μ_i and λ_i are the Lamé coeffcients and ρ_i the mass densities. The layered medium presents a triangular defect that extends within the thickness of the upper layer (Fig.1a). We restrict our interest to shear polarized surface waves propagating along the horizontal *x*-direction and polarized in the *y*-direction, known as Love waves. In the layer 1, the governing Helmholtz scalar equation reads:

$$\nabla_{(x,z)} \cdot \mu_1 \nabla_{(x,z)} u_y = \rho_1 u_{2,tt} \qquad (1)$$

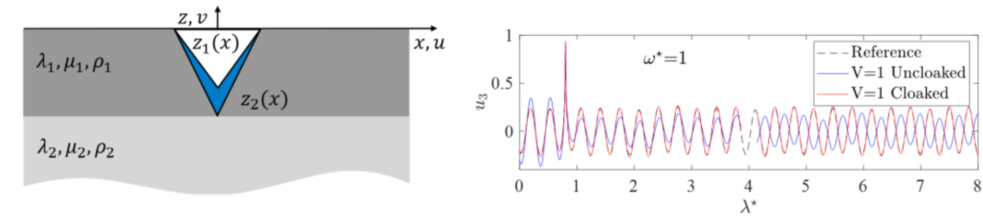
where u2 is the elastic displacement along y and u2,tt denotes the second order derivative in time of u2. Referring to Fig. 1 we introduce a change of coordinates $\{x, z\} \to \{u, v\}$, to shrink the white region of the triangular defect between $z = 0$ and $z = z_1(x)$ into the region between the curves $z_1(x)$ and $z_2(x)$ (blue region). Following the cloaking strategy in [2], we derive the governing equation in the transformed coordinate system, in a configuration similar to that in [3], as:

$$\nabla_{(u,v)} \cdot \mu'_1 \nabla_{(u,v)} u_y \quad = \rho'_1 u_{2,tt} \qquad (2)$$

where $\mu' = \frac{J_{ux} \mu J_{ux}{}^T}{\det(J_{ux})}$ and $\rho' = \frac{\rho}{\det(J_{ux})}$ are the transformed mechanical properties in the cloaked region (blue region in Fig 1a), and $J_{ux} = \frac{\partial(u,v)}{\partial(x,z)}$ is the Jacobian of the transformation.

Results

We model a triangular pinched cloak in a finite element environment using COMSOL Multiphysics. We performed time harmonic simulations and compared in Fig.1b the surface displacement field (z=0) for the (i) by-layer with no triangular defect (Reference), (ii) by-layer with the triangular defect without considering the elastodynamic transformation (Uncloaked), and (iii) the case of the cloaked defect (Cloacked). The results confirm that the out-of-plane displacement field of the cloaked defect configuration finely approximates the one of the pristine elastic media (i.e., no defect).



(a) **(b)**

Fig. 1. (a) Schematic of a Carpet-pinched cloak. (b) Reference, Uncloaked and Cloaked displacement field. λ^* is the wavelength at frequency ω^*, normalized with respect to the cut-of frequency of the first Love mode.

3. Concluding Remarks

We provided a brief theoretical account on the cloaking of a surface defect for elastic Love waves. An extension of the present approach to the cloaking of soft and rigid inclusions could pave the way towards the realization of novel isolating devices for out-of-plane surface elastic waves.

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