The effect of damping on the energy transfer in the spherical pendulum with fractional damping in a pivot point

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Abstract: Nonlinear vibrations of a system with three degrees of freedom with a spherical pendulum are investigated. The system contains an oscillator and a spherical pendulum suspended from the oscillator. The damping at the pendulum pivot point is assumed to be modelled by a fractional derivative. The viscoelastic damping properties are described using the fractional Caputo derivative of order $0 < \alpha \leq 1$. Vibrations in the vicinity of the internal and external resonance are considered. The effect of the order of the fractional derivative on the vibratiosn of the autoparametric system is studied. Responses of the system, the internal and external resonance, bifurcation diagrams, Poincaré maps and the Lyapunov exponents have been calculated for various orders of fractional derivatives. Chaotic motion has been found for some system parameters.

Keywords: spherical pendulum, fractional damping, nonlinear vibrations

1. Introduction

The impact of the fractional damping on dynamic properties of a coupled mechanical system with a spherical pendulum is investigated (Fig. 1). It is assumed that the spherical pendulum is suspended to the oscillator excited harmonically in the vertical direction $F_z(t) = P_1 \cos(\nu_1 t)$. The pendulum is excited harmonically in horizontal directions $F_x(t) = P_2 \cos(\nu_2 t)$, $F_y(t) = P_3 \cos(\nu_3 t)$. The oscillator contains a linear spring and a damper of a fractional type. Additionally, it assumed that damping moments acting in the pendulum pivot point are modelled by a fractional derivative. This study is an extension of the research presented by the authors in [1]

Assuming the fractional Caputo derivative defined as [2, 3]:

$$\frac{\mathrm{d}^\alpha}{\mathrm{d}t^\alpha} f(t) \equiv \dot{f}^{(\alpha)}(t) \equiv \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\frac{df(\tau)}{d\tau}\mathrm{d}\tau}{(t-\tau)^\alpha}, \ 0 < \alpha \leq 1 \tag{1}$$

where $\Gamma(1\text{-}\alpha)$ is the Euler gamma function, and $t > 0$.

Therefore, the dissipation force and moments are expressed as below

$$R(\dot{z}) = c_1 \dot{z}^{(\alpha_1)}, \quad M(\dot{\theta}) = c_2 \dot{\theta}^{(\alpha_2)}, \quad M(\dot{\varphi}) = c_2 \dot{\varphi}^{(\alpha_2)} \tag{2}$$

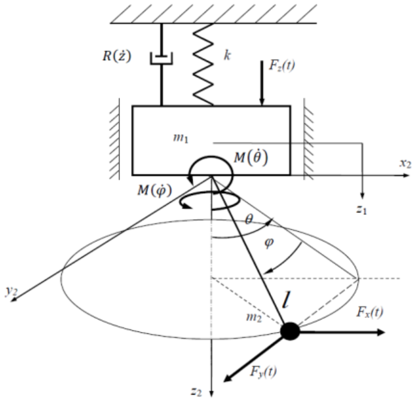

Fig. 1. Schematic diagram of the system

Using formulated above dissipation force and moments, the dimensionless equations of motion of the analysed system with spherical pendulum are as follows [1]

$$\ddot{z} - a\ddot{\theta}cos\phi sin\theta - a\ddot{\phi}\, sin\phi cos\theta = A_1 cos(\mu_1\tau) + a\big(\dot{\phi}^2 cos\phi cos\theta - 2\dot{\phi}\dot{\theta} sin\phi sin\theta -$$
$$\dot{\theta}^2 cos\phi cos\theta\big) - \gamma_1 \dot{z}^{(\alpha)} - z$$
$$\ddot{\theta}\cos^2\phi - \ddot{z}cos\phi sin\theta + \gamma_2\dot{\theta}^{(\alpha_2)} = 2\dot{\theta}\dot{\phi}cos\phi sin\phi - \beta^2 cos\phi sin\theta + A_2 cos\phi cos\theta \cos\mu_2\tau \quad (4)$$
$$\ddot{\phi} - \ddot{z}sin\phi cos\theta + \gamma_2\dot{\varphi}^{(\alpha_2)} = -\dot{\theta}^2 cos\phi sin\phi - \beta^2 sin\phi cos\theta - A_2 sin\phi sin\theta cos\mu_2\tau +$$
$$A_3 cos\phi cos\mu_3\tau$$

where

$$\omega_1^2 = \frac{k}{m_1+m_2} \quad \omega_2^2 = \frac{g}{l}, \beta = \frac{\omega_2}{\omega_1}, \gamma_1 = \frac{c_1}{(m_1+m_2)\omega_1^{2-\alpha_1}} \quad \bar{z} = \frac{z}{l}, a = \frac{m_2}{m_1+m_2}, A_1 = \frac{P}{(m_1+m_2)\omega_1^2 l} \quad \mu_1 =$$
$$\frac{\nu_1}{\omega_1}, \; A_2 = \frac{P_2}{m_2 l\omega_1^2}, \mu_2 = \frac{\nu_2}{\omega_1} \quad A_3 = \frac{P_3}{m_2 l\omega_1^2} \quad \mu_3 = \frac{\nu_3}{\omega_1}, \gamma_2 = \frac{c_2}{m_2 l^2\omega_1^{2-\alpha_2}} \quad (5)$$

3. Concluding Remarks

The influence of damping in the pendulum joint, described by fractional derivative on the energy transfer, internal and external resonances was studied. The performed calculations show that the vibration amplitude decrease with the increasing order of fractional derivative. Moreover, an increase in the derivative order causes shift in time the occurrence of energy transfer regions. It was shown that except different kinds of periodic vibrations there may also appear chaotic vibrations.

References

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