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Application of the Lumped-Parameter Method for Modelling Nonlinear Vibrations of Drill Strings with Complicating Factors

LELYA KHAJIYEVA[1], ASKAR KUDAIBERGENOV[2], YULIYA SABIROVA[3*]

Dept. Mathematical and Computer Modelling, Al-Farabi Kazakh National University, Almaty, Kazakhstan
[1. 0000-0002-2565-3409, 2. 0000-0001-9154-9653, 3. 0000-0002-3497-0940]
* Presenting Author

Abstract: This paper considers nonlinear spatial lateral vibrations of a drill string with supporting stabilizers in a gas flow. The mathematical model used for describing the studied process is based on Novozhilov's nonlinear theory of elasticity. The drill string lateral displacements are found by the lumped-parameter method (LPM). For numerical solution of the obtained discrete system, the fourth order Runge-Kutta method is utilized. The impact of the damping elements (supporting stabilizers) on the drill string lateral vibrations is analyzed, and numerical illustrations are presented for several cases.

Keywords: drill string, nonlinearity, lateral vibration, lumped-parameter method, stabilizers

1. Introduction

The search and application of the most effective approaches for modelling complex nonlinear processes, including poorly studied problems of dynamics of industrial equipment and machines at complicated conditions, are of large relevance. The purpose of this work is the assessment of the effectiveness of LPM for modelling the nonlinear dynamics of drill strings taking into account stabilizers and external forces. LPM is a special case of the finite element method, where the equation of a continuous medium is replaced by its discrete analogue, and is widely used in structural mechanics [1]. The influence of supporting stabilizers on the drilling system is under consideration due to the fact that accounting for damping elements can significantly improve the system stability and the efficiency of drilling and is of great scientific interest [2].

2. Mathematical Model

Based on the concepts of Novozhilov's nonlinear elasticity theory [3], a nonlinear mathematical model of spatial lateral vibrations of the simply supported rotating drill string in a gas flow accounting for external loads $N(x_3,t)$ and $M(x_3,t)$ [4] and the effect of stabilizers [2] is developed:

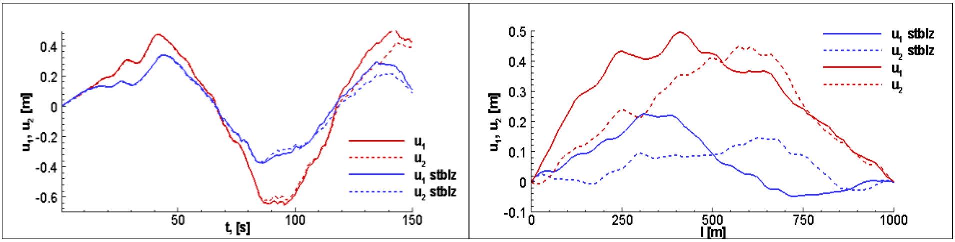
$$
\begin{aligned}
&\rho A\frac{\partial^2 u_1}{\partial t^2}+EI_{x_2}\frac{\partial^4 u_1}{\partial x_3^4}-\rho I_{x_2}\frac{\partial^4 u_1}{\partial x_3^2\partial t^2}+\frac{\partial^2}{\partial x_3^2}\left(M(x_3,t)\frac{\partial u_2}{\partial x_3}\right)+\frac{\partial}{\partial x_3}\left(N(x_3,t)\frac{\partial u_1}{\partial x_3}\right)-\frac{EA}{1-\nu}\frac{\partial}{\partial x_3}\left(\frac{\partial u_1}{\partial x_3}\right)^3\\
&-\frac{EA(5-6\nu)}{2(1-\nu)}\frac{\partial}{\partial x_3}\left(\frac{\partial u_1}{\partial x_3}\left(\frac{\partial u_2}{\partial x_3}\right)^2\right)-hP_0\kappa\left(\bar{M}\frac{\partial u_1}{\partial x_3}-\frac{\kappa+1}{4}\bar{M}^2\left(\frac{\partial u_1}{\partial x_3}\right)^2+\frac{\kappa+1}{12}\bar{M}^3\left(\frac{\partial u_1}{\partial x_3}\right)^3\right)\\
&-\rho A\left(2\Omega\frac{\partial u_2}{\partial t}+\Omega^2 u_1\right)+\sum_{j=1}^{J}\left(k_j^s u_1+c_j^s\frac{\partial u_1}{\partial t}\right)\delta\left(x-x_j^s\right)=0,
\end{aligned}
\tag{1}
$$

L. Khajiyeva: 71 Al-Farabi ave., 050040, Almaty, Kazakhstan, khadle@mail.ru.

$$\rho A\frac{\partial^2 u_2}{\partial t^2}+EI_{x_1}\frac{\partial^4 u_2}{\partial x_3^4}-\rho I_{x_1}\frac{\partial^4 u_2}{\partial x_3^2\partial t^2}-\frac{\partial^2}{\partial x_3^2}\left(M\left(x_3,t\right)\frac{\partial u_1}{\partial x_3}\right)+\frac{\partial}{\partial x_3}\left(N\left(x_3,t\right)\frac{\partial u_2}{\partial x_3}\right)-\frac{EA}{1-\nu}\frac{\partial}{\partial x_3}\left(\frac{\partial u_2}{\partial x_3}\right)^3$$
$$-\frac{EA(5-6\nu)}{2(1-\nu)}\frac{\partial}{\partial x_3}\left(\frac{\partial u_2}{\partial x_3}\left(\frac{\partial u_1}{\partial x_3}\right)^2\right)-hP_0\kappa\left(\bar{M}\frac{\partial u_2}{\partial x_3}-\frac{\kappa+1}{4}\bar{M}^2\left(\frac{\partial u_2}{\partial x_3}\right)^2+\frac{\kappa+1}{12}\bar{M}^3\left(\frac{\partial u_2}{\partial x_3}\right)^3\right) \qquad (2)$$
$$+\rho A\left(2\Omega\frac{\partial u_1}{\partial t}-\Omega^2 u_2\right)+\sum_{j=1}^{J}\left(k_j^s u_2+c_j^s\frac{\partial u_2}{\partial t}\right)\delta\left(x-x_j^s\right)=0.$$

3. Numerical Results

For obtaining solution of the mathematical model, LPM is utilized. According to this method, the given continuous equations (1), (2) are represented as a system of second-order ordinary differential equations by its numerical discretization at the end nodes. The numerical solution of the system is found by the fourth order Runge-Kutta method using the C++ programming language. The comprehensive analysis of the stabilizer influence on the drill string vibration process is carried out. It is shown that considering even one stabilizer allows substantially decrease the amplitude of the drill string vibrations in a gas flow (Fig. 1).



a) b)

Fig. 1. The stabilizer effect at $x_1^s = 0.6L$ on the drill string vibrations for a) $x_3 = 0.49L$, b) $t = 150\,\mathrm{s}$ $(\Omega = 0)$

4. Concluding Remarks

In this work, we considered the impact of stabilizers on the nonlinear dynamics of a drill string in a gas flow using LPM. According to the obtained results, the application of stabilizers allows considerably reducing the amplitude of the drill string vibrations. It proves the significance of studying the influence of damping elements on the drilling system and the efficiency of applying LPM when modelling vibrations of the drill string with complicating factors.

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