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# Stabilization of Course of Ships and Damping Vibrations Caused by Waves: Nonlinear Differential Equations Model and Optimal Control Theory

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**Abstract:** The paper is devoted to development of nonlinear differential equation model and construction of optimal control theory for stabilization of a ship's course and damping vibrations caused by waves of the sea. The suboptimal control law is calculated within a nonlinear model of a ship's motion.The whole system indludes the master differential temporal evolution equations for the angular velocity of a ship, an angle of rotation of the rudder, yawing angle $x_i$ . The solution is sought in the class of functions linear in $x_i$; accordingly, the control law has two modes: for small x, the control is formed in a linear mode, and for large x, it is formed in a relay mode. The problem of synthesis of the relay-linear law is reduced to the determination of the area D (where there is no vibration mode) and the synthesis of the control law *u=p(t)x*. The concrete numerical data are presented for optimal control theory of stabilization of a ship's course and damping vibrations caused by waves of the sea.

**Keywords:** optimal control, stabilization of a ship's course, damping vibrations

## 1. Introduction.

The methods of the theory of optimal control can be effectively applied to the problems of stabilizing the course of ships and damping vibrations caused by the waves of the sea. In this paper we develop a nonlinear differential equation model and construct an optimal control theory using an advanced method of synthesis of control for concrete dynamical system [1-3].

As an input model, one could choose the advanced model of a ship motion (e.g. details in Ref [1]). Let ω be the angular velocity of a ship around the vertical axis passing through the center of mass, and β is an angle of rotation of the rudder.

Then motion along the course can be described by the standard master differential equation as follows:

$$T_2 \frac{d^{2\omega}}{dt^2} + T_1 \frac{d\omega}{dt} \pm \omega + d_c \omega |\omega| = k_c \left( \beta + \tau_1 \frac{d\beta}{dt} \right) + \varphi(t) , \qquad (1)$$

where φ (t) is the disturbing force generated by the roughness of the sea, the plus sign in front of ω in this equation corresponds to a ship that is stable along the course, and the minus sign to an unstable one. Further let α be yawing angle, i.e.

$$d\alpha/dt=\omega. \quad (2)$$

One could suppose that a steering gear is described by the equation

$$\frac{d\beta}{dt}=\lambda\beta+bu \quad (3)$$

where $u$ is the control signal. The characteristic limitations on the steering angle β and its angle speed dβ/dt are usually as follows: β | ≤40 ° and | β | ≤5 deg / s. If one imposes a limitation on the input signal | u (t) | ≤1 and choose some realitic values for the parameters λ and b, then the indicated restrictions on β and dβ/dt will be satisfied for all t. Setting

$$x_1=\alpha, x_2=\omega, x_3=d\omega/dt\ x_4=\beta \quad (4)$$

and combining equations (1)-(3), one could obtain the system, which finally control of a ship's course and its stabilization. If the right part of such a system does not include $x_1$, therefore, the course control problem and the stabilization one at a given constant control can be described by the same model.

## 2. Results of Computing and Discussion

The numerical solution is sought in the class of functions linear in $x_i$; accordingly, the control law has two modes: for small x, the control is formed in a linear mode, and for large x, it is formed in a relay mode. The problem of synthesis of the relay-linear law is reduced to the determination of the area D (where there is no vibration mode) and the synthesis of the control law $u=p(t)x$. As the initial step, it is necessary to calculate a suboptinal relay-linear control law for a given dynamic system under condition φ (t)=0 in Eq.(1). The next step of computing includes the task with constantly acting perturbations and analysis of the corresponding relion. The wave disturbance is simulated by a linear system, which receives Gaussian white noise at the input. There are presented results of computing $x_i$ (i=1-4) for different sets of technical parameters. For example, the concrete data on the stabilization processes along the coordinates $x_1$ and $x_4$ are obtained numerically, the maximum deviation in the yawing angle at a given time delay is calculated, and and the effectiveness of the relion application for the problem of stabilizing the ship's course was shown.one of the important conclusions of computing is that a suboptimal in time control law has a high sensitivity to the variation of the right-hand sides of the differential equations that describe the movement of the ship along the course.

## 3. Concluding Remarks

The synthesis of an optimal in time control for complex dynamical system is considered and numerically solved on example of the problem of stabilizing the course of ships and damping vibrations caused by the waves of the sea.

## References


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